

result is known for groups G containing a dense image of \mathbf{R} (such groups are called *solenoidal*). The book gives corresponding results for \mathbf{Z}_p and \mathbf{Q}_p . A related problem is the following: Given an LCA group H , for what groups G are the continuous images of H dense in G ? Armacost investigates this question for various H . He also gives a brief discussion of the question of putting different topologies on a given Abelian group.

This book is one of the best organized that I have ever read. It begins with a careful selection of results assumed known to the reader (with references); the author refers to these results when needed, and he is unusually thorough in providing both these references and references to results previously proved. As a result, the book is a pleasure to read. There are only a few misprints and errors, and they should not cause any trouble; for instance in §4.27, p. 52, the reference should be to 4.25(a) rather than 4.26(a).

While the book does take up a great variety of topics, it can hardly be described as encyclopedic; after all, it is only 154 pages long. The author extends his coverage by concluding each chapter with a list of additional results, together with either a reference or a sketch of a proof. Still, many topics are slighted (as the author would no doubt admit); in particular, I wish that he had given more space to cohomological questions.

It would be unreasonable to expect mathematicians to buy this book in droves; the price is enough to inhibit most people. But many mathematicians may enjoy browsing through their library's copy of this pleasant and well-written book about an interesting and accessible subject.

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LAWRENCE CORWIN

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Between nilpotent and solvable, edited by Michael Weinstein, Polygonal Publishing House, Passaic, N.J., 1982, 240 pp., \$22.00.

In this book we find, in one volume, descriptions of many classes of finite solvable groups which include the nilpotent groups; supersolvable groups, M -groups, CLT-groups and related classes, linear groups, seminilpotent and

nilpotent-by-abelian groups, groups with the Sylow tower property, and several others. In addition, a chapter by Gary L. Walls presents the basic results on formations, Fitting classes, and homomorphisms; except for this chapter, all the classes of groups discussed are very special sorts, and the book must be viewed as a collection of special topics, not a collection of general results about solvable groups. Since the book is the combined effort of eight authors, some chapters, especially the one by Walls and the chapter on M -groups by B. M. Puttaswamaiah, are not very well related to the other chapters; but the authors write on their own favorite topics, and their enthusiasm is evident. The topics are not the subject of especially active current research; only 14 of the 84 research papers in the list of References are more recent than 1974. The chapter by Puttaswamaiah ends with a nice list of 18 exercises; only a few exercises and unsolved problems are mentioned elsewhere in the book.

The book's Preface, by the editor, briefly states some facts about nilpotent and solvable groups which the reader should know, and directs the reader to the 16 pages of Appendices, which present basic definitions and results on Philip Hall's extended Sylow theory, chief factors, covering and avoidance, Frattini subgroup, Fitting subgroups, the subgroups $O_p(G)$, $O_{p'}(G)$ and $O^p(G)$, the nilpotent residual, and the hypercenter.

Chapter 1, *Supersolvable groups*, consists of 42 pages by W. E. Deskins and Paul Venzke. I can best describe its contents by listing some of the main results, for a finite solvable G : (1) (R. Baer) G is supersolvable iff G has the Sylow tower property, and for every Sylow p -subgroup S of G , $N_G(S)/C_G(S)$ is strictly p -closed. (2) (Iwasawa) G is supersolvable iff G is equichained. (3) (Huppert) G is supersolvable iff every maximal subgroup has prime index. (4) (Agrawal-Kegel) G is supersolvable iff the hyper-generalized center of G is G . (5) If G has odd order, then G is supersolvable iff G satisfies the "permutizer condition"; each proper subgroup permutes with a cyclic subgroup that it does not contain. (6) G is supersolvable iff each maximal subgroup of G is weakly normal in G .

Much of the 26 pages of Chapter 2, *M-groups*, by B. M. Puttaswamaiah, presents the basic facts on representations, Clifford's theorem, monomial representations, and wreath products. G is an M -group if all its irreducible complex representations are induced from linear ones. The author proves Taketa's result that all M -groups are solvable, Dade's result that every finite solvable group is a subgroup of an M -group, and Huppert's result: if G has a normal solvable subgroup N whose Sylow subgroups are all abelian, and G/N is supersolvable, then G is an M -group. Of course, these results have been available in textbook form since 1967 (in B. Huppert's *Endliche Gruppen I*), but the discussion here is substantially different.

With the title *CLT and non-CLT groups*, the 47-page third chapter, by Henry G. Bray, could discuss any solvable groups; in fact, it discusses only a few classes of groups, some so special that they are completely classified. G is a CLT-group if the converse of Lagrange's theorem holds for G ; for every d dividing the order of G , G has a subgroup of order d . G is a BNCLT-group ("badly non-CLT") if G is not a p -group but every proper subgroup has prime power order. With two lengthy proofs, Bray classifies all BNCLT-groups (they

have order $p^a q$) and the orders of those nonsupersolvable groups with all proper divisors the orders of only supersolvable groups. The latter proof is not self-contained, since results of Pazderski are assumed.

Chapter 4, *Miscellaneous classes*, 27 pages by John F. Humphreys and David Johnson, first states but does not prove Suprunenko's description of primitive solvable linear groups. Then there are brief discussions of groups whose homomorphic images are all CLT-groups (QCLT-groups), groups which are the products of normal supersolvable subgroups, groups whose lattice of subgroups is lower semimodular (LM-groups), and seminipotent groups (those whose nonnormal nilpotent subgroups have nilpotent normalizers). Incidentally, the third G in the statement of Theorem 6.1, p. 137, should be a \mathcal{D} ; that was the most annoying of the several misprints I noticed in the book.

The title of Chapter 5, *Classes of finite solvable groups*, by Gary L. Walls, could title the entire book. This 44-page chapter actually is a well-written presentation of the standard results on formations and \mathcal{F} -normalizers due to Gaschütz, Lubeseder, Carter, and Hawkes, and the dual notion of Fitting class as developed by B. Fischer, Blessenohl, and Gaschütz. The last section, on the homomorph, a localized concept of formation developed by Wielandt, presents some results by J. A. Troccoli.

The last chapter, 19 pages by the editor, neatly summarizes much of the book by briefly stating or restating and proving known characterizations of certain classes of groups, as well as whether the classes are closed under the taking of subgroups, homomorphic images, or direct products. The classes are: CLT-groups, QCLT-groups, nilpotent-by-abelian groups, groups with the Sylow tower property, supersolvable groups, \mathcal{A} -groups ($G \in \mathcal{A}$ iff for all proper subgroups H , if prime p divides the index of H in G then there is a subgroup W in which H has index p), $\mathcal{S}^{\mathcal{A}}$ -groups ($G \in \mathcal{S}^{\mathcal{A}}$ iff all subgroups of G are in \mathcal{A}), and LM-groups. The omnibus Theorem 5.1 presents 23 equivalent conditions for supersolvability.

I would like to caution the reader that the list of References does not attempt to include all recent research papers on the topics of this book; it does, of course, include the results actually stated or referred to in this book. The index is also briefer than I would like. Since there does not seem to exist any other recent English language survey of special classes of solvable groups, this book should be a valuable addition to the libraries of those interested in the subject.

LARRY DORNHOFF

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Coding the universe, by A. Beller, R. B. Jensen, and P. Welch, London Mathematical Society Lecture Note Series, vol. 47, Cambridge Univ. Press, Cambridge, London, New York, New Rochelle, Melbourne, and Sydney, 1982, 353 pp., \$34.50.