he did write. But, as I've indicated, I don't think much of it, so I've tried to express some of the excitement I feel about research in this field. The real danger in writing a book is that you can turn people off and conceal the enthusiasm that workers in the subject have. I hope I've counteracted that a little.

REFERENCES

1. C. Bessaga and Ed Dubinsky, *Nuclear Fréchet spaces without bases*, Arch. Math. **31** (1978), 597–604.

2. Manfred Börgens, Reinhold Meise and Dietmar Vogt, *Functions holomorphes sur certains espaces échelonnés et λ-nuclearité*, C. R. Acad. Sci. Paris (to appear).

3. L. Crone and W. B. Robinson, *Every nuclear Fréchet space with a regular basis has the quasi-equivalence property*, Studia Math. **52** (1975), 203–207.

4. M. M. Dragilev, *On regular bases in nuclear spaces*, Amer. Math. Soc. Transl. **93** (1970), 61–82.

5. Ed Dubinsky, *Subspaces without bases in nuclear Fréchet spaces*, J. Functional Anal. **26** (1977), 121–130.

6. ______, *The structure of nuclear Fréchet spaces*, Lecture Notes in Math., vol. 720, Springer-Verlag, Berlin and New York.

7. Ed Dubinsky and Boris Mityagin, *Quotient spaces without bases in nuclear Fréchet spaces*, Canad. J. Math. **30** (1978), 1296–1305.

8. A. S. Dynin and B. S. Mityagin, *Criterion for nuclearity in terms of approximative dimension*, Bull Acad. Polon. Sci. Sér Sci Math. Astronom. Phys. **8** (1960), 535–540.

9. P. Enflo, *A counter example to the approximation problem in Banach spaces*, Acta Math. **130** (1973), 309–317.

10. A. Grothendieck, *Produits tensoriels topologiques et espaces nucleaires*, Mem. Amer. Math. Soc. No. 16 (1955).

11. G. Henkin and B. S. Mityagin, *Linear problems of complex analysis*, Uspehi Mat. Nauk. **26** (4) (1972), 95–152. (Russian)

12. T. Kōmura and Y. Kōmura, *Über die einbettung der nuklearen Räume im* $(s)^A$, Math. Ann. **162** (1966), 284–288.

13. B. S. Mityagin, *Approximative dimension and bases in nuclear spaces*, Uspehi Mat. Nauk. **16** (4) (1961), 73–132. (Russian)

14. ______, *Equivalence of bases in Hilbert scales*, Studia Math. **37** (1971), 111–137. (Russian)

15. B. S. Mityagin and N. M. Zobin, *Contre-exemple á l'existence d'une base dans un espace de Fréchet nucleaire*, C. R. Acad. Sci. Paris Sér. A. **279** (1974), 255–256; 325–327.

16. A. Pietsch, *Nukleare Lokalkonvexe Räume*, Berlin, 1965.

17. D. Vogt, *Subspaces and quotient spaces of* (s), Functional Analysis, Surveys and Recent Results, (Proc. Conf., Padenborn, 1976), North-Holland, Amsterdam,1977, pp. 167–187.

18. V. P. Zahariuta, *On the isomorphism of Cartesian products of locally convex spaces*, Studia Math **46** (1973), 201–221.

19. ______, *Generalized Mityagin invariants and a continuum of pairwise nonisomorphic spaces of analytic functions*, Funkcional Anal. i Priložen **11** (1977), 24–30. (Russian)

ED DUBINSKY

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 3, Number 1, July 1980

0002-9904/80/0000-0316/$02.00

Mathematical modelling techniques, by R. Aris, Research Notes in Mathematics, no. 24, Pitman, London-San Francisco-Melbourne, 1979, x + 191 pp., $15.00.

Forming and studying mathematical models and talking about forming and studying mathematical models are both quite fashionable right now. While the activity of model building (the forming and studying) has no doubt been practiced for hundreds, if not thousands, of years, most descriptions of this activity seem to have been written during the last decade. These descriptions tend to fall into one of two classes: textbooks that illustrate the activity primarily by considering many examples with references and exercises ([2], [4], [7], [8], [11], [13]) and short articles which contain relatively brief descriptions of the methodology of model building with relatively few examples or exercises (see [5], [6] and articles in [10] and [14]). The monograph by Blalock [3] is an exception since it examines the methodology of model building in some detail. *Mathematical modelling techniques* by Aris is also an exception. It is intended to be a practical guide to the craft of model building. There are no exercises, and the examples used are quite specialized. On the other hand, the discussion is quite general in nature; and an attempt is made to consider all aspects of the model building process.

To describe the art of model building is, in a sense, an attempt to define the notion of a mathematical model. However, since a definition which is given by a lengthy description is not satisfactory to everyone, Aris also gives a direct definition very early in his text. It states that a mathematical model is:

> "any complete and consistent set of mathematical equations which is thought to correspond to some other entity, its prototype".

This definition is one that many individuals who are interested in model building will be comfortable with. Indeed, these individuals view mathematical models as sets of equations. On the other hand, there are also many individuals who consider the notion of a mathematical model to be considerably more general than the one described by Aris. There is no doubt, however, that in all cases the term mathematical model is intended to refer to something which is expressed in mathematical terms and which is, in some way or other, related to a nonmathematical entity (the prototype of Aris). There also seems to be little debate about the fact that most mathematical models are subject to evolution, and they evolve best in an environment which includes the study of the appropriate nonmathematical entity. In other words, mathematical models are not only associated with and affected by developments in mathematics, but they also influence and are influenced by developments in some branch of the sciences.

As an example of an area of study in which there are models which seem to be mathematical in nature, but which are not sets of equations, we mention the area of preference theory and voting. Arrow's basic work [1] is the most well known, but there has been and continues to be a great deal of other activity in the field ([9], [12], [15]). The models in this field continue to evolve (much like more traditional models in the physical sciences), yet they are not easily characterized by a system of equations.

Since forming useful mathematical models is a process which is much like that of developing useful mathematical theories, it is not easy to prescribe a formula for doing it. There are, however, certain steps in the process which seem to be followed by most practitioners of the art. These include simplification of the real setting or problem, replacement of the real objects and relationships by mathematical objects and relationships, study of the resulting mathematical structure, and (when possible) comparison of the conclusions and/or predictions obtained from the mathematical study with hard data. An illustrious example of this process and of its evolutionary nature is provided by the study of planetary motion. The study and the mathematical models go back at least 2000 years to proposals made by the Greeks, and they extend through Newton's Laws of Gravitation to Einstein's Theory of Relativity.

Within the very broad framework noted above, there are many tricks of the trade which will aid the model builder. Keeping in mind that in general Aris has restricted his discussion to mathematical models which are systems of equations, we now consider the topics covered in his description and discussion of model building.

Using his definition as a starting point, Aris devotes much of Chapter one (What is a model?) to a discussion of the many and varied uses of the terms "model". This is an excellent survey of this topic. Anyone seriously interested in it would be well advised to begin here. However, it should also be noted, that Chapter one, despite its excellent survey, does have one serious shortcoming. The examples used here, and throughout the text, are very specialized ones. To be specific, the examples, which are described in detail in the appendices, are: Longitudinal diffusion in a packed bed, The coated tube chromatograph and Taylor diffusion, and The stirred tank reactor. There is no doubt that these examples do illustrate the ideas discussed by the author. The problem lies in their lack of appeal to the casual reader who though interested in models may well not be particularly interested in these examples. At this stage the goals of the author might have been better served by a number of more elementary examples.

In Chapter two (The different types of models) the author acknowledges the lack of breadth in his choice of examples, and he provides a listing, with very brief descriptions, of many other types of models now used in applications. It would be difficult for a novice to learn much about any of these models from the short descriptions given here. However, the chapter does serve to show that many quite different types of models are now used and that they are used in a wide variety of disciplines. Also, the references which are cited will enable a puzzled reader to find out much more about the models cited. Topics surveyed include: graphs, game theory, linear programming, fuzzy sets, statistical models, difference and differential equations, and stochastic models.

Chapter three (How to formulate a model) begins with one of the more surprising statements in the book. Namely, "Comparatively little needs to be said on this score now that we have reviewed the types of models that are available for the formulation is nothing more than a rational accounting for the various factors that enter the picture in accordance with the hypotheses that have been laid down". Thanks to a series of examples this preliminary statement is quickly shown to be a substantial understatement of the difficulties involved in formulating models. These examples are all models formulated with partial differential equations, and they are based, in the authors words, on laws and conservation principles. Unfortunately, it is just such formulations which are so difficult for a beginner in model building.

Chapter three also contains a section which discusses the alternative formulation of discrete and continuous models of physical problems. This is a valuable section as it illustrates some of the difficulties involved in going from one of these types of model to the other.

After a mathematical model is formulated it must often be manipulated so that it will be in a form which provides (as readily as possible) the information sought by the model builder. Since the desired form is usually not known in advance by the model builder, a period of experimentation is almost inevitable. In Chapter four the author has provided his version of a set of guidelines for model builders to use in manipulating their models. As examples of such guidelines we note that the first two (of thirteen) are: to cast the problem in as elegant a form as possible and to choose a sympathetic notation, but don't become too attached to it. One of the techniques of manipulation which is described in some detail is that of rendering the variables and parameters dimensionless. That such a rendering is a useful tactic is not obvious, and the author makes a valuable contribution by pointing out why it might help and how to do it.

In addition to his guidelines for manipulating models the author also points out the value of somehow getting a feel for the solution without doing a tremendous number of detailed calculations. Again, guidelines are given; and it is noted that no particular method is preferable in all cases and (at least in this sort of a search) the end justifies the means.

Study of a mathematical model includes evaluating the model. The fifth and final chapter of the text considers this important aspect of the model building process. In particular, the following topics are discussed: effective presentation of a model, extensions of models, observable quantities, and comparison of models and prototypes and of models among themselves. The discussion here is quite specific and a number of examples are considered.

To conclude, Aris has made an important and a valuable contribution to the literature on model building. One would hope for more such efforts. Interest is high, and there is always more to say on such a topic.

BIBLIOGRAPHY

1. K. J. Arrow, *Social choice and individual values*, 2nd ed., John Wiley, New York, 1963.

2. E. A. Bender, *An introduction to mathematical modelling*, John Wiley, New York, 1978.

3. H. M. Blalock, Jr., *Theory construction: from verbal to mathematical formulation*, Prentice-Hall, Englewood, N. J., California, 1969.

4. R. Haberman, *Mathematical models: mechanical viberation, Population dynamics, and traffic flow*, Prentice-Hall, Englewood Cliffs, N. J., 1977.

5. J. M. Hammersley, *Maxims for manipulators*, Bull. Inst. Math. Appl. **9** (1973), 276–280.

6. ______, *Poking about for the vital juices of mathematical research*, Bull. Inst. Math. Appl. **10** (1974), 235–247.

7. C. C. Lin and L. A. Segal, *Mathematics applied to deterministic problems in the natural sciences*, MacMillan, New York, 1974.

8. D. P. Maki and M. Thompson, *Mathematical models and applications: with emphasis on the social, life and management sciences*, Prentice-Hall, Englewood Cliffs, N. J., 1973.

9. N. R. Miller, *Logrolling, vote trading, and the paradox of voting: a game theoretic overview*, Public Choice **30** (1977), 51–75.

10. The National Research Council's Committee on Support of Research in the Mathematical Sciences, eds., *The Mathematical Sciences*, M.I.T. Press, Cambridge, Mass., 1969.

11. M. Olinick, *An introduction to mathematical models in the social and life sciences*, Addison-Wesley, Reading, Mass., 1978.

12. W. H. Riker and S. J. Brams, *The Paradox of vote trading*, American Political Science Review **67** (1973), 1235–1247.

13. F. Roberts, *Discrete mathematical models: with applications to social biological, and environmental problems*, Prentice-Hall, Englewood Cliffs, N. J., 1976.

14. T. L. Saaty and F. J. Wegl, *The spirit and the uses of the mathematical sciences*, McGraw-Hill, New York, 1969.

15. E. M. Uslaner and J. R. Davis, *The paradox of vote trading: effects of decision rules and voting strategies on externalities*, American Political Science Review **69** (1975), 929–942.

DANIEL P. MAKI

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 3, Number 1, July 1980

0002-9904/80/0000-0317/$01.50

Mathematics of finite-dimensional control systems. Theory and design, by David L. Russell, Lecture Notes in Pure and Applied Mathematics, Vol. 43, Marcel Dekker, New York and Basel, 1979, viii + 553 pp., $45.00.

Control theory was brought into existence during the second half of the eighteenth century by the development of complex machinery such as the steam engine. Since that time until about 1900 it was primarily concerned with elimination of undesirable traits (chiefly instability) by means of feedback devices, the Watt governor being a notable example; design was mainly the result of intuition and empirical insights. The beginning of the theory can be traced to J. C. Maxwell's celebrated paper on governors [1]. Progress was slow during the nineteenth century but became faster after 1900 due to the development of power transmission, communications and complex processing plants and some mathematical techniques (such as the Routh-Hurwitz stability criteria) began to be systematically used. Growth was enormous during and after the second World War and many other mathematical tools like Laplace transforms and probability theory found applications. In the late fifties and early sixties, starting with the work of Bellman, Glicksberg and Gross [2], Bellman [3], Pontryagin, Boltyanskiĭ, Gamkrelidze and Mischenko [4], Kalman [5], Kalman and Bucy [6] and others, control theory began to be accepted as a respectable mathematical discipline. It also started to absorb relatively sophisticated "modern" mathematics into its language (for instance measure theory, elementary functional analysis, abstract algebra and Liapunov stability theory) and brought to the forefront the idea of *quality of control*: if the control engineer was content in the past, say, with rendering stable the operation of a machine by means of a feedback device his modern counterpart would try to achieve the same effect in a suitably optimal way (for instance, minimizing the stabilization time, the cost of the control device, the strain on the machinery, etc.). Finally, concepts like controllability, observability and stabilization by feedback, until then living in a latent state in the literature were given precise formulations.

Although many of the initial contributions to the mathematical theory of control were firmly rooted in reality (for instance, the influence of [3] and [6] in modern technology was and is enormous) control theory tended to develop along two parallel lines since the early sixties. The first is practiced by