are K-valued, K a local field. Their work establishes that there is a lot of interesting mathematics in this area.

This current contribution is a text book and is extra-ordinarily complete, in the areas with which it deals: An introduction to requisite valuation theory and topology; Banach spaces (spherical completeness, orthogonal bases); Banach algebras; Integration; Invariant ("Haar-like") measures; and a brief look at the Fourier theory. It is filled with exercises, outlines of unsolved and partially solved problems and excellent notes. The author makes no attempt to be encyclopedic; avoiding topics such as: special functions, categories of Banach spaces, analytic functions and rings of power series. He does, however, give some references to the literature for those topics.

The list of references is long and overly complete in some ways and is lacking in others. On the plus side we have that the notes and comments are well and specifically referenced. However, the introduction refers to some work that is not referenced and an occasional reference seems to be included because it has the right words in the title and not because it is used or refers to any subject discussed.

The fashionable view is that if functional analysis is not Archimedean then it is either a trivial extension of the Archimedean case which holds because of certain abstract nonsense, or it is trivial because it clearly fails or holds for the most elementary of reasons. Consider some counter-examples: If G is a nice enough non-Archimedean group and $\hat{G}$ is its dual (by the way, there are *three* natural notions of dual available) then the space of finite measures (non-Archimedean valued measures!) on G is isomorphic with space of bounded uniformly continuous functions on $\hat{G}$. For Banach algebras, the rather spectacular failure of the Gelfand-Mazur theorem leads to *four* non-equivalent notions of the spectral radius, and it would seem that the future holds, not a general theory of Banach algebras, but a variety of such theories.

If you have the patience to read a text book, and it takes patience to read a text, you will find that the view that non-Archimedean functional analysis is trivial is not entirely correct.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 1, Number 5, September 1979

0002-9904/79/0000-0417/$01.75

Quasi-ideals in rings and semigroups, by Ottó Steinfeld, Akadémiai Kiadó (Publishing House of the Hungarian Academy of Sciences), Budapest, 1978, xii + 154 pp.

The notion of a quasi-ideal was first introduced by the author, for rings in 1953 and for semigroups in 1956. An additive subgroup Q of a ring A is called a *quasi-ideal of* A if $QA \cap AQ \subseteq Q$. The same definition applies if A is a semigroup, changing "additive subgroup" to "non-empty subset". More than fifty papers have appeared since that time, dealing with quasi-ideals, and the present book gives a systematic survey of the main concepts and results in this area. The author himself is the outstanding contributor, with more than a dozen papers on the subject.

A subset of a semigroup S is a quasi-ideal of S if and only if it is the intersection of a left ideal of S with a right ideal of S. The same is true for a ring A with identity, and would be true in general if we modified slightly the definition of a quasi-ideal Q of A by requiring that $(Q + QA) \cap (Q + AQ) \subseteq Q$ (hence $= Q$), instead of $QA \cap AQ \subseteq Q$. All the statements in this book remain true for this slightly narrower concept.

The main part of the book consists of twelve sections, not grouped into chapters. This is followed by an appendix, entitled *Quasi-absorbents in groupoid-lattices*, consisting of five sections. In the main part, the ring and semigroup theory are developed side by side, so to speak, while in the appendix the two theories are brought under a common proof.

One of the interesting features of the book is the constant comparison of the two theories. One is surprised at how similar they can be in some respects, and how different in others. For example, the class of regular rings (in the sense of von Neumann) without nonzero nilpotent elements can be characterized by any one of the following six equivalent conditions on a ring A. (These rings were characterized as regular subdirect products of division rings by A. Forsythe and N. McCoy in 1946.)

(1) A is a regular ring in which every one-sided ideal is two-sided.

(2) A is regular, and $x \in Ax^2A$ for all $x \in A$.

(3) For any left ideal L and any right ideal R of A,

$$RL = R \cap L \subseteq LR.$$

(4) For any left ideal L and any right ideal R of A,

$$R \cap L = LR.$$

(5) For any two quasi-ideals Q_1 and Q_2 of A,

$$Q_1Q_2 = Q_1 \cap Q_2.$$

(6) For any quasi-ideal Q of A,

$$Q^2 = Q.$$

These conditions are taken from Theorem 9.7, 9.10, 11.1, and 11.5. The first three of these theorems apply to semigroups as well as rings, and combine to show that if we replace "ring" by "semigroup", the six conditions fall into two sets of mutually equivalent ones:

Group I: (1), (4) (5);

Group II: (2), (3), (6).

Each condition in Group I [II] characterizes the class of semilattices of groups [regular simple semigroups]. These are worlds apart: a regular simple semigroup is a far cry from being a group!

Another good feature of the book is a list of twenty-one open problems. Here is one the reviewer found intriguing. The intersection of a [0-] minimal left ideal with a [0-] minimal right ideal of a ring A [semigroup S with zero] is either 0 or a [0-] minimal quasi-ideal of A [S] (Theorem 6.1). Question: for what rings A [semigroups S with zero] is every [0-] minimal quasi-ideal of A [S] the intersection of a [0-] minimal right ideal with a [0-] minimal left ideal of A [S]? This holds if A [S] is semiprime, i..e, contains no nonzero nilpotent ideals (Theorem 7.2), but not in general. In addition to these open problems, the final section of the main part of the book is devoted to various generalizations and further directions of research.

More than half of the book is concerned with [0-] minimal ideals, and with decompositions of rings [semigroups] into sums of such. For example, Theorem 8.1 asserts the equivalence of the following four conditions on a ring A.

(A) A is semiprime and the sum of its minimal left ideals.

(B) A is the direct sum of two-sided ideals B_λ $(\lambda \in \Lambda)$ each of which is a simple ring containing at least one minimal left ideal.

(C) A is the sum of a complete system of quasi-ideals.

(D) A is semiprime and the sum of its minimal quasi-ideals.

(It was shown independently by C. C. Faith in 1959, and F. Szasz in 1960, that each B_λ in (B) is isomorphic to a dense subring of the ring of linear transformations of finite rank of a vector space over a division ring.) In (C), a *complete system of quasi-ideals* is a set $\{Q_{\gamma\delta}: \gamma \in \Gamma, \delta \in \Delta\}$ where $Q_{\gamma\delta}$ is either $\{0\}$ or a minimal quasi-ideal of A of the form $e_\gamma A f_\delta$, with e_γ and f_δ idempotents, satisfying a further condition (involving orthogonal idempotents) too complicated to repeat here. The corresponding class of semigroups is that of primitive regular semigroups, and Theorem 10.1 is the analogue of Theorem 8.1. Here the notion of complete system of quasi-ideals is much simpler; we can take $\Gamma = \Delta$, and the "further condition" is just that $Q_{\gamma\delta} \neq 0$ implies $Q_{\gamma\delta}Q_{\delta\gamma} \neq 0$.

The appendix presents a common generalization of the two theories of quasi-ideals. It is in the same spirit as other "abstract" theories of ideals, initiated by Dedekind himself. (See, for example, the chapter on lattice-ordered monoids in Garrett Birkhoff's *Lattice theory*, third edition, 1967.) A groupoid-lattice is a complete lattice $V(\wedge, \vee)$ endowed with a further binary operation $(\cdot)$ satisfying:

(1) $a \leqslant b \Rightarrow a \cdot c \leqslant b \cdot c$ and $c \cdot a \leqslant c \cdot b$, for all $a, b, c \in V$;

(2) $a \cdot a \leqslant a$, for all $a \in V$;

(3) $a \cdot 0 = 0 \cdot a = 0$, for all $a \in V$, where 0 is the least element of the lattice V.

If $a,b \in V$ and $b \leqslant a$, then b is called a *left* [*right*] *absorbent of* a if $a \cdot b \leqslant b$ $[b \cdot a \leqslant b]$, and a *quasi-absorbent of* a if $a \cdot b \wedge b \cdot a \leqslant b$.

If A is a ring, the groupoid-lattice $V(A)$ of A is the lattice of subrings of A, with the product $X \cdot Y$ of two subrings X, Y of A defined to be the smallest subring of A containing $\{xy: x \in X, y \in Y\}$. The groupoid-lattice $V(S)$ of a semigroup S with zero is defined similarly, with "subring" replaced by "subsemigroup with 0". In both cases, product so defined is in general non-associative. The notions of left, right, and quasi-absorbents evidently become those of left, right, and quasi-ideals in $V(A)$ and in $V(S)$. The appendix culminates in a decomposition theorem which generalizes Theorems 8.1 and 10.1 mentioned above.

The book is very well written, the English excellent, the proofs full and clear. It is attractively printed, with very few misprints. The reviewer found it a pleasure to read.

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