$n_1 = n_2 = 4$  is also analyzed. There is now an extra isomorphism corresponding to the graph automorphism of the Dynkin diagram for  $C_2$ .

Much of the material in this book, including the two theorems above, is more general than previously published results in the area. Recently, Callan [2] has applied the same method to unitary groups over noncommutative domains possessing a division ring of quotients, assuming the underlying Witt index is at least three. The objectives of the book and the basic methods are very clearly presented. No problems are included, but then none are needed, for the best way to fully understand some of the more intricate proofs is to break them into pieces, as does the author, and work out the separate details for oneself.

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Fundamentals of decision analysis, by Irving H. LaValle, Holt, Rinehart and Winston, New York, Chicago, San Francisco, Atlanta, Dallas, Montreal, Toronto, London, Sydney, 1978, xiii + 626 pp.

To know the rules is not the same as to know how to play the game. As in chess or tennis, so in decision analysis. Decision analysis is applied decision theory, or how to make decisions that are consistent with the choices, information, and preferences of the decision maker. Decision analysis is both a language and philosophy for decision making and a practical procedure for arriving at decisions. The procedure consists of analyzing (Latin: loosening back) the decision problem into its choice, information, and preference component parts, which can then be judgmentally assessed by the decision maker and combined by logic to allow a consistent course of action. To see why this book would be more appropriately titled *Fundamentals of statistical decision theory*, we must consider the present state of decision analysis in more detail.

The domain of decision analysis is shown graphically in Figure 1. The first three rows represent the three elements of formulation that we have dis-

cussed. The last row represents the analysis or logic used to arrive at a course of action. The columns represent the three levels of application: the formal theory, the procedures that constitute the methodology, and the professional practice or discipline. In the legal arena, for example, these could correspond to the knowledge of law, the procedure of filing a lawsuit, and the management of a case for a client.

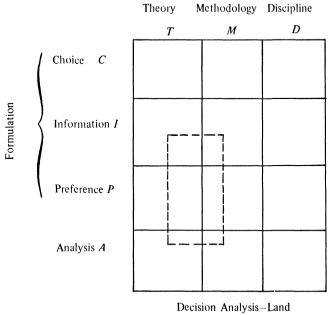


Figure 1

The current state of the art in decision analysis covers the entire figure, but not equally well. For example, the treatment of choices is the weakest area in decision analysis; we know little more than laymen about how to create new alternatives either in theory or in practice.

However, the balance of the figure is remarkably well-understood at all levels though the profession still has much to learn. There are now dozens of decision analysts who earn their living by performing decision analysis for business and government.

We can now begin our discussion of Professor LaValle's book. The primary coverage of the book is shown by the dotted region in the figure; theory is emphasized over practice.

Let us first discuss what is common to decision analysis and to the book. Any discussion of decision analysis must spend most of its time on the assessment of uncertainty by means of probability and on the assessment of risk attitude by means of utility functions. The heart of the book-the first six chapters—is concerned with just these issues and the representation of the problem in decision tree form. The "rollback" of decision trees (which would occupy the AT box of our figure) is the main analytical concept introduced. My impression of this section is that the main ideas could have been presented with considerably more efficiency. There is no concept here that

cannot be conveyed to an intelligent layman in a day; I have seen it done.

Much of the space is devoted to presenting probability theory, but too lightly to develop an appreciation of the subject. For example, change of variable is treated in an exercise, yet it is one of the most perplexing of subjects for the student. It is inconceivable to me that someone who can comprehend change of variable in such terse form would not be bored by the discursive presentation of decision trees. This is just one example of what I regard as a "moving target" problem in defining the book's intended readership.

The remainder of the book is devoted to such topics as decisions in normal form (of little practical interest), statistical inference (standard Bayesian treatment), Markovian decision processes (succinct, clear), group decision problems (interesting presentation of limited scope), and game theory (standard, introductory). I question whether game theory is properly part of decision analysis, though it is an interesting subject.

Now we shall turn to aspects of decision analysis that are treated little, if at all, in the book. We begin with philosophical issues. The argument for using these methods for decision making is not strongly advanced. The Allais paradox, which challenges the paradigm and demands an answer (and there are several), is presented as an exercise with a few sentences of discussion. The students for whom the book is intended surely deserve to be aware of the kind of behavior consistent with and inconsistent with the axioms.

Another philosophical shortcoming is the inadequate treatment of the entire judgmental position, including the important contributions of Jeffreys, Cox, and Jaynes. Both decision making and inference rest on axiomatic foundations that are considerably stronger in their appeal to the intuition than the axioms of probability.

Finally, and perhaps most important, the cornerstone of decision analysis, the distinction between decision and outcome, is never explicitly made. This principle has been widely recognized as forming a basis not only for decision analysis, but for mental health. To put it even more plainly, without decision analysis we have no way to define what is meant by a "good" decision.

Continuing now to practical omissions, there is no discussion of the role of modeling in decision analysis. Anyone who tries to apply decision theory to a practical problem will soon see the importance of modeling. People have difficulty assessing the plethora of probabilities and values required for an analysis without assistance from structural representation. Only to a decision theorist does the world seem simple enough to be treated by direct assessment alone.

But even if we try to rely on pure assessments, we are ill-prepared to undertake them by the treatment in the book. The many pitfalls of probability and risk attitude assessment pointed out by Kahneman and Tversky and treated by methods developed in the SRI Decision Analysis Group are nowhere described. Performing decision analysis without careful attention to assessment is comparable to carelessness in preparing the input to a computer program: garbage in, garbage out.

Finally, there is no discussion of the many tricks of the trade that make decision analysis possible in a world of busy people. The reader will not learn

here about simple, practical methods for bounding problems, for determining sensitivities, for encoding probability, or for approximating risk preference.

And so we may ask again, for whom is the book written? In the preface, we find that it should be suitable for business schools and for economic and applied mathematics courses at the upper undergraduate and lower graduate levels. The author has doubtlessly thought about some of the problems that will arise when a typical business school student encounters a Jacobian determinant. This may explain why he has made Appendix 1 a chart of the Greek alphabet. However, I think that the problem lies at a deeper level. For business school students I find the book too mathematical relative to the insights it produces. For the mathematically inclined student, it seems to avoid many problems of real mathematical interest, such as how to assess uncertain functions as well as uncertain variables. For the practical student, such as the engineer, it falls short in presenting the links between theory and practice that are essential in application.

Thus, what might have been an interesting mathematical text on statistical decision theory falls somewhat short of its title as a treatment of decision analysis. The book is an extensive, meticulous, and well-written elementary text on decision theory for the mathematically inclined, but it is not an effective guide to either the philosophical comprehensiveness or professional practice of decision analysis.

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Groupes et anneaux réticulés, by Alain Bigard, Klaus Keimel and Samuel Wolfenstein, Lecture Notes in Math., vol. 608, Springer-Verlag, Berlin and New York, 1977, xi + 334 pp.

The book under review is a study of groups and rings which carry a (necessarily distributive) lattice structure in such a way that the group operation distributes over the lattice operations, and in addition, in the case of rings, products of positive elements are positive.

The study of algebraic systems where an order relation is introduced, compatible with the algebraic operations, does not have a long history. While various algebraic generalizations of the real number field have been the subjects of extensive theories in the 19th century (quaternions, matrix and linear algebras etc.), the importance of order relation in algebra has been completely overlooked. The explanation for this might be found in the absence of total order in the complex number field and in the old opinion that inequalities serve to express continuity and as such they are alien to algebra.

Towards the end of the last century, the necessity of order relation in algebraic systems emerged in the foundations of plane geometry (D. Hilbert): the collection of coordinates on the line had to be made into a totally ordered field. In this respect, a decisive role was played by the archimedean axiom which was listed by Hilbert (along with the completion axiom) among the