In order to explain the applications to Markov processes it is necessary to extend the theory to trajectories in the space of distributions (in the sense of probability theory) induced by the semigroup of a Markov process–for example, Brownian motion. For measure dynamics thus extended, the objects of interest become positive contractions of L_1 instead of measure-preserving transformations of the underlying space. The proof of the maximal ergodic theorem may then be viewed as a statement about the potential theory, the balayage and the stopping times of the underlying Markov process. There are many other results as well which unite measure dynamics, potential theory and the theory of Markov processes. Regrettably, Sinai's book includes no discussion of these ideas, though it would have been quite possible to develop them in keeping with the spirit of the book, by considering special cases–the random walk, for example.

Professor Sinai's lectures are beautifully written. Our criticism may be summarized by saying simply that they end too soon. We hope that Professor Sinai will publish a sequel adding problems which will illustrate more clearly the mathematical applications of ergodic theory and which will go further in developing the theory in general terms.

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Vorlesungen über numerische Mathematik, by Heinz Rutishauser, Birkhäuser Verlag, Basel, Switzerland, Bands 1 and 2, 1976, 164 pp. and 229 pp., Fr/DM 40,48.

The two volumes under review here are elementary lecture notes on numerical analysis written by Heinz Rutishauser before his premature death in 1970 at the age of fifty-two. Although Rutishauser intended ultimately to publish these notes as a textbook, they were by no means in final form at his death, and in spite of the able editorship of Martin Gutknecht they remain somewhat rough-hewn and not a little out of date. Nonetheless, Rutishauser was one of the most successful and respected workers in this field, and it is not surprising that his notes represent one of the best introductions to numerical analysis as it is actually practiced.

The first thing to remark about these notes is that they are thoroughly modern in spirit, reflecting the changes that the computer has brought to the field. Numerical analysis is an old subject; indeed because pre-Greek mathematicians were accustomed to casting their problems in numerical form, it is difficult to separate it from the origins of mathematics itself. Many great mathematicians have been interested in the art of computation: Newton, Lagrange, and especially Gauss are notable examples. However, it was von Neumann who initiated the modern period of numerical analysis by his fathering of the digital computer.

The effect of the computer has not been merely quantitative; it has changed the way people think about numerical algorithms. In the first place, the high speed and large storage capacities of modern computers have made practical algorithms that would have been dismissed as impossible in the days algorithms practical that would have been dismissed as impossible in the days of hand computation. Conversely, many algorithms designed with hand human can quickly scan an array of numbers and determine the largest, a computer must proceed by individual comparisons.

The algorithms Rutishauser chooses to describe were clearly meant to be implemented on a modern computer. For example, he avoids the extended treatment of the difference calculus, which for a time was obligatory in books on numerical analysis. As pretty as this subject is, it has little to do with modern numerical computations, and Rutishauser rightly develops that small part en route to the useful interpolation formulas.

A second effect of the computer has been to place new emphasis on the careful design of algorithms. One has only to attempt to program an algorithm of moderate complexity to realize that an outline in mathematical notation is not sufficient to produce an effective implementation. A commonplace example is the quadratic formula, whose naive use can give unnecessarily inaccurate results. In matrix computations one must pay careful attention to memory management, since most computers provide storage to handle no more than a few matrices of order, say, two hundred.

Rutishauser was a master of the art of algorithmic design, and this shows throughout his notes. In the first place, the algorithms are clearly motivated so that the reader can see their insides. Second, they are accompanied by carefully chosen numerical examples. Some of these simply illustrate the normal behavior of an algorithm. However, many of them probe the pathology of a method, and these are the most illuminating of all. For it is only by seeing repeatedly how good ideas can go wrong that one can get an intuitive grasp of the difference between a good and a bad algorithm. Finally, Rutishauser treats in detail questions of implementation on a computer, even to the point of including program fragments.

The computer has also increased the need for careful analyses of numerical algorithms. In hand computation numerical instability often exhibits itself in an obvious way, and even if the person performing the computation cannot understand exactly what has gone wrong, he is at least free to try something else. Programming such decisions into a digital computer may be difficult or impossible, and there is now a decided preference for algorithms whose properties are so well understood that they can be expected to work on the class of problems for which they were intended.

I say expected rather than proved, because it is seldom that a numerical algorithm can be analyzed in a completely rigorous manner. The example of Gaussian elimination for the solution of linear algebraic equations may make this clearer. In a now classical rounding-error analysis, J. H. Wilkinson has shown rigorously that Gaussian elimination with partial pivoting is numerically stable, provided no growth of elements occurs in the course of the reduction. However, he also has exhibited a matrix for which large elements do occur; consequently, the final error bounds contain a factor of 2^n, which makes them unusable in practice.

This negative result notwithstanding, much has been gained from the analysis. A welter of algorithmic detail has been simplified so that one can decide the question of stability by examining a single number, the growth factor g mentioned above. This result can be used in various ways. First, it suggests means, e.g. complete pivoting, of improving the stability of the algorithm. Second, one can attempt to prove that for certain classes of matrices g cannot become large, and this has been successfully done. Third, one can monitor g during the computations and report failure if it is so indicated. Finally, one can perform experiments to determine empirically if g is likely to be large. This last approach has turned out to be the most fruitful; the largest value of g reported for any matrix that was not specifically designed to make the algorithm fail is twenty-three. For this reason Gaussian elimination with partial pivoting is the usual way of solving a general system of linear equations.

Many analyses of numerical algorithms are like this. A rigorous mathematical analysis simplifies the details of an algorithm to the point where one can determine rationally how it will behave in practice. In this connection it is important to analyze the algorithm at hand, not one modified to fit the mathematical tools at one's disposal. It is also important to answer the right questions. For example, I have seen an elaborate analysis establishing the ultimate swift convergence of an iterative method that in practice can grind interminably before its asymptotic behavior is realized. In any case, these constraints make the mathematical analysis of numerical algorithms an extremely challenging occupation.

Rutishauser gives as much analysis as the elementary nature of the notes and the need for an uncluttered exposition permit. He presents an interesting analysis of how to determine in the presence of rounding error if a matrix is positive definite. In his discussion of the stability of methods for solving initial value problems he uses the common device of analyzing a model problem that is simple enough to be tractable and typical enough to be useful. And there are his ubiquitous numerical examples to supplement the analyses.

These are the major virtues of the notes. The details are as follows.

The two volumes cover a variety of topics. The first volume treats the solution of linear systems of equations and least squares problems, the solution of nonlinear equations, interpolation, numerical integration, and approximation. The second volume treats the numerical solution of ordinary and partial differential equations and the algebraic eigenvalue problem. An appendix presents advanced material relating to the *qd* algorithm for computing zeros of polynomials and includes an elegant axiomatic treatment of computer arithmetic.

Since the notes were intended for elementary courses, they presuppose little mathematical background on the part of the reader. A thorough familiarity with calculus and elementary linear algebra is all that is required for most of the work. However, this does not greatly limit its scope, since most numerical algorithms can be derived from rather elementary considerations, even though a complete analysis may require a great deal of mathematical apparatus. Moreover, the elementary approach has enabled the author to segregate his topics into essentially independent essays. This is no doubt more the result of the lecture note format than of design, but I find the style quite congenial to the eclectic nature of numerical analysis.

As might be expected from the circumstances of their publication, the notes are uneven, with some parts having more polish than others. More seriously, much of the work is out of date. No mention is made of the use of finite elements to solve partial differential equations; nor is the QR algorithm mentioned in the sections on algebraic eigenvalue problems. I found myself wishing that the editor had appended annotated references to more recent works. Not only would this have increased the value of the notes, but it also would have reduced the chances of the casual reader's being misled about the current state of the art.

However, the virtues of the work far outweigh its defects. It is unfortunate that it is available only in German; for it deserves to be more widely read. Rutishauser's audience is not only the student, but the instructor teaching a numerical analysis course for the first time, and especially the mathematician who wants to find out what this important branch of applied mathematics is all about.

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Lie groups and compact groups, by John F. Price, London Mathematical Society Lecture Note Series, no. 25, Cambridge Univ. Press, Cambridge, London, New York, Melbourne, 1977, ix + 177 pp., $8.95.

There are few truly successful classification theorems in mathematics–that is, theorems which describe all examples of an apparently large class of objects in a relatively simple and constructive way. One of the best such theorems classifies compact connected Lie groups. As is usual in the subject, I shall use the word "simple" to mean what is also called "almost simple": G is simple if it has a finite center C such that G/C has no nontrivial closed normal subgroups. Now let G be any compact, connected Lie group, let Z be the connected component of the identity in the center of G, and let $H = [G, G]$, the closure of the commutator subgroup of G. The classification theorem says that Z is (isomorphic to) a torus and that $G \cong Z \times H/F_0$, where F is a finite central subgroup. Moreover, $H \cong G_1 \times \cdots \times G_n/F$, where $G_1, \ldots, G_n$ are simply connected simple Lie groups (uniquely determined up to order by H) and F is a finite central subgroup. Finally,