want to prove that one Hamiltonian circuit implies at least 3 in a trivalent graph? He does it by a piece of algebra. Does he want a condition for a graph to have a perfect matching? (See Chapter 5.) He extracts it from an identity involving Pfaffians. Does he want to enumerate planar maps of some kind? He solves functional equations for formal power series. And it is not just a matter of one worker's inclinations. Look at the towering structure of general graphical enumeration theory! It is built of permutation groups and their cycle indices, and its pinnacles are formal power series [**6**].

So at times I gaze into the Future and contemplate a Mathematics in which there is no Graph Theory. That has been absorbed into Linear Algebra [**5**], or perhaps the Theory of Formal Power Series. But this mood does not last, since I am naturally optimistic. My vision usually ends with a glorious resurrection in the form of Matroid Theory.

I like matroids. I think of them as combinatorial objects of the same general kind as graphs, – generalizations of graphs in fact, – and even more desirable because they always have duals. It is true that I am not yet very good at drawing them, and if I thereby stand convicted of the feminine weakness of illogicality, then so be it. Matroid Theory brings with it out of the sea of Algebra "the abstract properties of linear dependence" and we discover paradoxically that fundamentally linear dependence is not an algebraic concept at all, even if it is at times decorated with fields and rings.

But I am getting ahead of my subject. Matroids are not discussed in Bondy and Murty. Still, we can always hope for a sequel, or an expanded Second Edition.

Meanwhile the present work gives us Graph Theory in its state of purity. It is really an outstanding book. Why, Appendix III alone (Some Interesting Graphs) is worth "a thousand pounds a puff".

REFERENCES

1. Blanche Descartes, *A three colour problem*, Eureka, April 1947 (solution, March 1948).
2. ______, *Network-colourings*, Math. Gaz. **32** (1948), 67–69. MR **10**, 136.
3. ______, *Solution to problem* 4526, Amer. Math. Monthly **61** (1954), p. 352.
4. ______, *The expanding unicurse*, Proof techniques in graph theory (Proc. Conf., Ann Arbor, 1968) (E.F. Harary, editor), Academic Press, New York, 1969, p. 25. MR **40** #4162.
5. ______, *Song of the subgraph hunter*, Combinatorial Structures and Their Applications (Proc. Internat. Conf., Calgary, 1969), Gordon and Breach, New York, 1970, p. vi.
6. ______, *Enumerator's song*, Combinatorial Structures and Their Applications (Proc. Internat. Conf., Calgary, 1969), Gordon and Breach, New York, 1970, p. vi.
7. ______, *On a theorem of Kuratowski*, Discrete Math., Master Index Issue (Vols. 1–10), 1975.
8. William Shakespeare, *As you like it*, Act II, Scene one.

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Asymptotic wave theory, by Maurice Roseau, Applied Mathematics and Mechanics, vol. 20, North Holland/American Elsevier, Amsterdam/New York, 1976, x + 349 pp.

The beauty of wave motion has long fascinated mankind even though he may not always have been aware that he was observing waves in action. The charming ripples on a pond and the fiercesome motions of the sea are plain for all to see but the azure sky, the glorious sunset, the twinkling star and the brass band are not at first recognised as wave phenomena. In fact, there are few areas of explanation of natural events which do not rely on waves whether one be concerned with molecule, jet noise, the vagaries of the weather or the esoterics of general relativity. That most have to wait until university before realising the ubiquity of undulations is an illustration that the beautiful is mysterious and not easily explicable. It is one thing to perceive the attraction of an exciting occurrence and quite another to make reliable quantitative predictions of the subsequent behaviour.

Two categories of waves may be distinguished theoretically–the linear and the nonlinear. Probably there are no real waves which are truly linear but many practical situations exist in which a linear model is reasonable. Linearised equations have been studied extensively and are appreciably simpler than the nonlinear yet the simplest demands mathematics of university level. Only in recent years have analytical and numerical techniques been devised that render the nonlinear problem tractable.

The great advantage which linearised equations enjoy is the availability of the principle of superposition so that the linear combination of two solutions is also a solution. A simple concept, yet its implementation has led to Fourier series and thereby to general expansions in eigenfunctions. But there are many situations where such expansions are not profitable so integral representations were adopted and the theory of integral equations evolved. Powerful methods of approximating integrals then become necessary: so the method of stationary phase and asymptotic expansions are born. The very diversity of wave phenomena has enriched the tree of analysis and encouraged it to spread branches which have become growths in their own right. The researcher in the linear theory ought to have a better than passing acquaintance with the analytical methods and cannot afford to despise numerical techniques either. For many common occurrences such as the rainbow still defy a genuine quantitative explanation. Much remains to be done to improve weapons before it can be asserted that the scattering of waves (whether elastic, electromagnetic, water or other) by an obstacle (whose shape and properties are freely chosen by the investigator) can be fully elucidated. Nevertheless this must be one objective of the applied mathematician if he is to meet the requirements of the modern engineering designer.

The writer on waves faces the formidable task of selecting some of the waves in the linear shop and examining them fully or of glancing briefly at all of them for he cannot hope to incorporate in a single display all of the products each of which could justify a whole shelf devoted to it. However, should he be disposed to present only those items which are also relevant to the nonlinear situation his choice will be severely limited. A perhaps natural progression would be to turn to nonlinear functional analysis. But, despite many high-powered efforts, the main achievements of nonlinear functional analysis have been in the demonstration of the existence of bifurcations where instabilities often arise. So it will be more profitable for the writer to pick on the theory of rays and associated variational principles. Rays are uniquely valuable in the asymptotic investigation of both linear and nonlinear waves.

It is therefore natural on opening a book on Asymptotic Wave Theory to enquire what aspects of rays are covered. The reader will seek in vain, for there is not a single mention of ray. He will find the Laplace transform, Bessel functions and the method of steepest descent for integrals described and then applied to various problems in water waves and seismology. But the nearest he will get to a ray is in one short section on characteristics. The absence of one of the most powerful modern tools for evaluating the asymptotic performance in many different physical contexts is a serious deficiency in a book purporting to deal with asymptotic theory. Whatever other methods are eliminated in the process of selection this one must not be discarded. The applied mathematician of today dare not be ignorant of rays which can offer a viable approach both analytically and numerically when other techniques are hopeless. The book at his elbow and the book he shows his students need to tell the reader about the propagation of energy along rays, transport equations and Hamilton's principle. Without these topics the value of a book is that much less. Caveat emptor.

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Stochastic processes in queueing theory, by A. A. Borovkov, Applications of Mathematics, no. 4, Springer-Verlag, New York, Heidelberg, Berlin, 1976, xi + 280 pp., $29.80.

Queueing theory is that branch of applied mathematics which attempts to construct and analyse models for what might be called 'unpredictable congestion'. There are many practical situations in which 'customers' demand some sort of 'service' which they cannot immediately obtain because of the demands of other customers. Very often the congestion is caused by variability, in the arrival pattern of the customers, or in the service mechanism, or both, and any model must be expressed in terms of random processes, and can be expected to yield conclusions in probabilistic language.

The early development of the theory was motivated by the problems of congestion in telephone systems, first in Scandinavia (A. K. Erlang) and later in the United States and France (F. Pollaczek). At first it grew in isolation from other manifestations of applied probability, but gradually the connections with the growing theory of random processes came to be realised and exploited. In the West this process may be said to have been completed in 1951 when D. G. Kendall addressed a famous meeting of the Royal Statistical Society, but in Russia the work of A. Ya. Hincin had by then already introduced the subject to the thriving Russian school of probabilists.

It must be admitted that the last quarter-century has been more notable for quantity than for quality of published research. It is too easy to devise a slightly different queueing system and to study it by what are now standard methods. If one's results can be kept safely under cover of several Laplace transforms, they are safe from comparison with reality. And indeed, those