indicates, but primarily numerical methods for partial differential equations. Modern methods, for example, finite elements, fast Fourier transform, and the method of large particles (particle-in-cell method) are discussed or mentioned, while a large part of the book is devoted to the powerful splitting-up method. This method is based on the formal relations

$$e^{(A_1+A_2)\Delta t} = e^{A_1\Delta t}e^{A_2\Delta t} + O(\Delta t)^2$$

and

$$e^{(A_1+A_2)\Delta t} = e^{A_1\Delta t/2}e^{A_2\Delta t/2}e^{A_2\Delta t/2}e^{A_1\Delta t/2} + O(\Delta t)^3$$

which permit the problem $u_t = (A_1 + A_2)u$ to be solved as a sequence of simpler problems.

I do find one fault with this book. It paints too rosy a picture of computational physics. Only the most serendipitous practitioner will be able to use successfully some of the recommended methods on complicated problems. A better balance would have resulted with the inclusion of a chapter on ways of analyzing the effectiveness of a scheme; phase error analysis, operation counts, long-time stability properties, and detailed truncation error analysis. Such a chapter might also have included some numerical results to show the bad answers that some apparently good methods can produce.

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Funktionalanalysis, by Harro Heuser, Mathematische Leitfäden, B. G. Teubner, Stuttgart, 1975, 416 pp.,

Geometric functional analysis and its applications, by Richard B. Holmes, Graduate Texts in Mathematics, No. 24, Springer-Verlag, New York, Heidelberg, Berlin, 1975, x + 246 pp., $16.80.

Functional analysis, by Michael Reed and Barry Simon, *Methods of modern mathematical physics*, vol. I, Academic Press, New York and London, 1972, xvii + 325 pp., $13.50.

Methods of modern mathematical physics, vol. II, *Fourier analysis, self-adjointness*, by Michael Reed and Barry Simon, Academic Press, New York, 1975, xv + 361 pp., $24.50.

These are three quite different introductions to functional analysis, addressed to different constituencies; all three are intended for use as graduate level textbooks, with varying demands on the reader's mathematical background. Heuser's book is appropriate for general mathematics students as well as future specialists. Holmes' book stresses Banach spaces and applications to optimization theory. Reed and Simon's series (apparently projected for at least five volumes) is an exposition of functional-analytic methods in modern mathematical physics. In different ways, these books are all written admirably, but I confess that for sheer craftsmanship and pedagogical judgement, my heart belongs to Heuser.

Reed and Simon. Volume I is a straightforward mathematics text, intended as a two-semester introduction to 'applicable' functional analysis. Except for a three-page concluding section on mathematical problems of quantum mechanics, there is very little explicit physics in this volume. (Chapter headings: I. Preliminaries; II. Hilbert spaces; III. Banach spaces; IV. Topological spaces; V. Locally convex spaces; VI. Bounded operators; VII. The spectral theorem; VIII. Unbounded operators.) The reader is expected to be familiar with metric space topology and with integration theory (Lebesgue and abstract); this material is reviewed in Chapter I, with numerous examples but almost no proofs. The reader is also expected to have some prior acquaintance with the material in Chapters II–IV. The level of exposition is uneven. If the proof of a theorem is brief and elementary, it is likely to be included; if it is long or difficult or technical or messy, it is likely to be omitted. {For example, Chapter IV contains a lightning tour of general topology. It is proved that a continuous bijection between compact spaces is a homeomorphism, but Tychonoff's theorem on product spaces and Urysohn's theorem are stated without proof. The same chapter contains an outline of integration theory on a compact space X; it is proved that a positive linear form on $C(X)$ is bounded, but the Riesz-Kakutani representation theorem is stated without proof.} What is uniform is that definitions and propositions are well motivated, stated clearly, and well illustrated with examples. If the reader can absorb mathematical ideas without going through the drudgery of formal proofs, this book will be a useful guide. Because of the excellent motivational discussions and the general banishment of pain, the book is fun to browse through. Notable inclusions: distributions, unbounded operators. Notable omissions: Banach algebras, the Kreĭn-Milman theorem (apparently these will be taken up in Chapters XV and XVI). The spectral theorem is confined to self-adjoint operators (with many details left as exercises). Volume II consists of two massive chapters (IX. The Fourier transform; X. Self-adjointness and the existence of dynamics). The Fourier transform is described for distribution spaces and L^p-spaces, and applications to the solution of partial differential equations are discussed. Symmetric and self-adjoint operators are explored in depth, as well as the generation of semigroups of operators, with applications to equations of physics (heat equation, Schrödinger's equation, wave equation, etc.). Throughout Volume II, the physics flies thick and fast, and, from the beginning, overwhelmed this reviewer. The mathematics, too, is sophisticated; altogether, a formidable jump between Volumes I and II. The entire series as projected is awesome in scope, suggesting that it will become a 'Dunford-Schwartz' of mathematical physics. (Forthcoming chapter headings include Perturbations of point spectra, Scattering theory, Spectral analysis, Group representations, Von Neumann algebras, Applications to quantum field theory, Applications to statistical mechanics.)

Holmes. The author's intent, in his own words, was to produce "a source of functional analytic information for workers in the broad areas of modern optimization and approximation theory", and it is his hope that the book will also serve the needs of students intending to specialize in research in Banach space theory. Fair enough. The author pursues his objective with ruthless professionalism, circling it several times over. By "geometric" functional analysis, he means that the spotlight is on the concept of convexity. This is borne out by the chapter headings: I. Convexity in linear spaces (45 pp.); II. Convexity in linear topological spaces (73 pp.); III. Principles of Banach spaces (83 pp.); IV. Conjugate spaces and universal spaces (33 pp.). In the author's view, the Hahn-Banach theorem is the single most important general principle of geometric functional analysis (ten equivalent versions are presented), and the second most important is the Kreĭn-Milman theorem. Conspicuously absent are spectral theory (drummed out in the Preface as irrelevant to geometric functional analysis) and Banach algebras. (The author therefore has no need for complex function theory: general topology and integration theory are the operative prerequisites.) Indeed, since convexity is the principal hero, most of the drama unfolds in settings no richer than a Banach space. The emphasis is on real scalars. {For instance, despite the ten equivalent formulations of the Hahn-Banach theorem, it was not clear to the reviewer that the complex version of the theorem (i.e., the extension due to Bohnenblust and Sobczyk) is ever proved. It is cited in 16D on p. 123; the reference there leads back to 16C, then to 11G, then to 6A, and ultimately to Exercise 1.21, which pertains to real linear spaces (at which point I gave up the search).} The author's preoccupation with convexity occasionally produces grotesque results. {His "geometric" proof of the Stone-Weierstrass approximation theorem employs the Hahn-Banach theorem, the Alaoglu-Bourbaki theorem, the Kreĭn-Milman theorem, and the Riesz-Kakutani theorem (whose proof in turn uses the Hahn-Banach theorem and the extremal disconnectedness of the Stone-Čech compactification of a discrete space); as technique runs amok, some may say that they really understand the Stone-Weierstrass theorem for the first time, but others will weep silently for Stone's beautiful proof.} Real scalars emphasized, ring structure nearly ignored, Hilbert spaces entirely ignored, spectral theory omitted; one is reminded of W. Rudin's illuminating review of Banach's masterpiece [see W. Rudin, *Functional analysis*, pp. 372–373, McGraw-Hill, 1973], to which Holmes's book is a worthy heir. One cannot fault the author that his book gives a view of functional analysis less representative of the state of the subject than did that of Banach; the author has named candidly the audience to which he is writing, he proves the hard theorems of his subject (including, for example, the theorems of Eberlein-Smulian and James on weak compactness), and he delivers the applications promised in his Preface (systems of linear inequalities, best approximation, control theory, convex programming, error estimation, splines, variational inequalities, etc.). Precisely because it is so successful in meeting the needs of its specialized audience, Holmes's book is unsuitable for general audiences. {For an introductory course in functional analysis for general mathematics students, I recommend either the book of Rudin cited above, or, for those who can navigate the German, the book of Heuser.}

HEUSER. There are 14 chapters, organized into 110 sections. The format, selection of topics, and didactic qualities reminded me of Riesz and Sz.-Nagy's masterpiece. Chapter headings (translated freely):

I. Banach's fixed-point theorem
II. Normed spaces

III. Dual vector spaces and adjoint operators
IV. Fredholm operators
V. Four principles of functional analysis (Hahn-Banach theorem, Baire category theorem, open mapping and closed graph theorems, principle of uniform boundedness)
VI. The Riesz-Schauder theory of compact operators
VII. Spectral theory in Banach spaces and Banach algebras
VIII. Approximation problems in normed spaces
IX. Orthogonal decomposition in Hilbert spaces
X. Spectral theory in Hilbert spaces
XI. Topological vector spaces
XII. Locally convex spaces
XIII. Duality and compactness
XIV. The representation of commutative Banach algebras.

This is a book for beginners that gets somewhere. The prerequisites are modest. The author makes do with metric topological notions for the first 3/4 of the book (§1 is a concise introduction to metric spaces); general topological notions are deferred until §81, just before they are needed for the discussion of weak topologies. Complex function theory is needed in Chapter VII (for Gelfand's theory of spectrum in commutative Banach algebras, and for the holomorphic functional calculus). The Lebesgue integral is not needed for reading the text (but applications involving it are sketched in the exercises). The contents of Chapters I–VII are amply suggested by the chapter headings. Chapter VIII discusses approximation in strictly convex spaces, uniformly convex spaces and Hilbert spaces. In Chapter IX one finds the Riesz-Fréchet theorem, orthonormal bases, and adjoint operators. In Chapter X, the spectral theorem is proved for compact normal operators and for bounded self-adjoint operators. Chapter XI begins with a section on metric vector spaces; a section on general topological spaces prepares the way for weak topologies and then general topological vector spaces. Chapters XII and XIII cover the standard lore of locally convex spaces and duality theory, including the theorems of Hahn-Banach, Alaoglu-Bourbaki, Kreĭn-Milman, Mackey-Arens; a special bonus is a proof of the Schauder fixed-point theorem. The concluding Chapter XIV is devoted to commutative Banach algebras with unity, including the Gelfand representation theorem and the Gelfand-Naĭmark characterization of the algebras $C(X)$, X compact, as the commutative C^*-algebras with unity. A fitting end to a stimulating voyage.

All of these books are attractively printed. All have extensive problem lists. Reed and Simon do their referencing as they go along, mostly in the notes at the end of each chapter, thus there is no bibliography at the end of either volume; Heuser gives unobtrusive references throughout the text, to an extensive bibliography (176 items) at the end of the book; Holmes's book ends with a brief guide to the literature and a bibliography of 81 items.

CONCLUSIONS. Reed and Simon, written for a special constituency, stands in a class by itself. Holmes's book is the most important contribution to mathematics. Heuser's book is the most important contribution to mathematical pedagogy; the reviewer believes that an English translation would prosper.

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