reading. The mathematician not working in lattice theory can get from this book a good idea of the history and importance of distributive lattices and their role in logic and universal algebra. The book is also suitable as a text for graduate students and the numerous exercises scattered throughout should be quite helpful especially to the novice doing independent reading. In two areas the reviewer wishes things might have been different. The authors use $+$ and $\cdot$ instead of $\vee$ and $\wedge$ throughout; one's preference here is somewhat a matter of "creature comfort" and it must be said that the authors are in good company with respect to their notation (see for example von Neumann's *Continuous geometry*). In a text such as this one, designed to bring the reader to the frontiers of current research, it would have been a natural thing to include in addition to the exercises some specific open problems. Although it is to be hoped that any such collection will soon become out of date, the inclusion of such problems does give a feeling for what the experts are asking and sometimes provides impetus and a challenge, especially for graduate students.

In summary, *Distributive lattices* is worth having. It was carefully planned and well written, providing a survey of the general area, special topics, and information on where to find more. It is a useful reference work for lattice theorists and a good source of information for those not conversant with the field, where perhaps it can kindle a spark of interest in the position and development of one of lattice theory's oldest branches.

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MARY KATHERINE BENNETT

BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 2, March 1976

Pseudo-differential operators, by Michael Taylor, Lecture Notes in Mathematics, No. 416, Springer-Verlag, Berlin, Heidelberg, New York, 1974, iv+155 pp., $7.40.

The reverse of differentiation is integration; the reverse of a linear partial differential operator is, most likely, some kind of integral operator, such as the Newtonian potential. The classical operators of potential theory have been generalized in stages, first to singular integral operators, then to pseudo-differential operators, and on to Fourier integral operators. The heart of these theories is a "*functional calculus*". The singular integral operators, for instance, are mapped homomorphically onto a class of functions, called "symbols" of the operators: The composition of operators corresponds to the pointwise product of symbols, adjoints to complex conjugates, and sums to sums. The homomorphism has a kernel which, in the classical applications, consists of "negligible lower order terms". The one-dimensional case arose in boundary problems for the Laplace and Cauchy-Riemann equations, and was developed by Hilbert, Plemelj, Noether, Riesz, Carleman, et al. The n-dimensional case, arising again with the Laplace equation, was developed from the 30's through the 50's, mainly by Giraud, Mikhlin, and Calderón-Zygmund.

The theory took a new turn when Calderón exported these methods from their native territory, and applied them to the Cauchy initial value problem. Singular integrals allowed him to reduce general equations to first order and then diagonalize, just as in the theory of ordinary differential equations. A second important application was made by Atiyah and Singer in the index problem for elliptic operators; singular integrals gave them the topologist's freedom to deform the general case into basic special cases. Another geometric application was the generalization by Atiyah and Bott of the Lefschetz fixed point theorem.

The next advance was motivated by the "$\bar{\delta}$-Neumann boundary problem" in several complex variables. This problem is similar to the ones considered by Hilbert, et al., but now the lower-order terms cannot be neglected. Kohn and Nirenberg refined the theory to take account of these terms, and gave their theory the name "pseudo-differential operators". Hörmander made further generalizations, aimed at "hypoelliptic" problems: When does $Au \in C^\infty$ imply that $u \in C^\infty$?

The notes by Michael Taylor present, essentially, Hörmander's version, with "symbols of type $S^m_{\rho,\delta}$". They include a good selection of basic applications: elliptic and hypoelliptic equations, hyperbolic operators, elliptic and parabolic boundary problems, and finally, wave front sets, with Hörmander's theorem on the propagation of singularities along characteristics. The "sharp Gårding inequality" of Hörmander, Lax-Nirenberg, and Friedrichs is proved and used; other fine points of the theory, such as the Calderón-Vaillancourt theorem on L^2 norms, and its application, appear only in the bibliography.

The topics are efficiently presented, but the style is brisk, even brusque. It is worth while to read the sources indicated by Taylor, to flesh out his basic outline, or to see how the Deus got in the machina.

Good exercises are always welcome; these notes have a lot, explicitly labeled, and some of the familiar implicit exercises: "correct the sign", "add the missing term", "fill the gap", "find the missing hypothesis". The book will be useful for a graduate course or seminar; or for individual study, given enough strength and sophistication in analysis. A nodding acquaintance with partial differential equations would help too; the author does not take time to introduce them, except for a brief discussion of the vibrating membrane.

[I am afraid that some of the remarks in this review may reveal my ignorance. It would be comforting to discuss them with the principals; but I cannot even check them against the crucial articles, which are unavailable within several thousand miles... Caveat lector.]

R. T. SEELEY