is the binomial expansion of the function (in powers of z^{-1})

$$h(z)=\sum_{j=1}^{t}\left[\sum_{k=1}^{m_j}\frac{A_{jk}}{(z-y_j)^k}\right],"$$

It seems the numerator should be $A_{jk}z^k$ and that the proof must therefore be altered. The proof of the parabola theorem on p. 51 is not correct.

The book reflects the author's love and enthusiasm for the subject. It surely will be an important reference text in the field for years to come for physicists, engineers, chemists and mathematicians, pure and applied.

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BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 2, March 1976

Distributive lattices, by Raymond Balbes and Philip Dwinger, University of Missouri Press, Columbia, Missouri, 1975, xiii+294 pp., $25.00.

Lattice theory, as an independent branch of mathematics, has had a somewhat stormy existence during its hundred-odd years of being. Its origins are to be found in Boole's mid-nineteenth century work in classical logic; and the success of what we now call Boolean algebra in this field led to the late nineteenth century attempts at the formalization of all of mathematical reasoning, and eventually to mathematical logic.

Schröder and Peirce introduced the concept of an abstract lattice as a generalization of Boolean algebras, while Dedekind's work on algebraic numbers led him to the introduction of lattices outside of logic and to the concept of modular lattices. These late-nineteenth century investigations did not lead to widespread interest in lattice theory—it was not until the thirties that lattice theory truly became an object for independent and systematic study by mathematicians.

Stone's representation theory for Boolean algebras and distributive lattices, Menger's work on the subspace structure of geometries, von Neumann's coordinatization of continuous geometry and Birkhoff's recognition of the lattice as a basic tool in algebra were among the forces which combined in the late thirties to enable Birkhoff successfully to promote the idea that lattice theory is a branch of mathematics worthy of the attention of the community.

The very simplicity of the basic concepts in lattice theory and the degree of abstraction in its relationship to other branches of mathematics have proved to be at once both its strongest and weakest points.

Lattices are ubiquitous in mathematics. The beauty and simplicity of the abstraction and the ability to tie together seemingly unrelated pieces of mathematics are certainly appealing to the mathematician-as-artist. The introduction of new and nontrivial techniques for the solution of outstanding problems, for example in universal algebra, is mathematically rewarding; and the discovery of new questions which become natural to ask in the context of lattice theory is undoubtedly intriguing.

Through the vehicle of lattice theory one can hope to contribute to progress in functional analysis by the study of vector lattices or the investigation of von Neumann algebras. Closely related is the work that has been done in quantum mechanics and the special partially ordered sets and lattices that arise there. A different type of work in the field is that being done in combinatorial theory where geometric lattices have been used extensively by Rota and his school in the investigations inspired by Whitney and begun by Dilworth.

These are but a few of the avenues of development of general lattice theory since the surge of interest in it forty years ago. During these ensuing years the sailing was not always smooth. Since lattice theory many times does not provide answers to questions in other fields it is often considered too abstract to be really useful. Together with category theory it suffers from the fact that abstraction many times leads to oversimplification and loss of the basic properties of the underlying structure. Perhaps even more unfortunate is that the simplicity of the basic definitions leads to the production of an inordinate amount of printed trivia. While the appearance of trivia is not foreign to any branch of mathematics, that which arises in lattice theory is quite easily recognizable as such by nonexperts in the field. An obvious result of this phenomenon is that the entire field at times becomes discounted as a serious branch of mathematics.

The role played by distributive lattices in the development of lattice theory is both an important and an interesting one. Boolean algebra and modular lattices have been with us since the last century. Since distributive lattices are weaker than the former and stronger than the latter, they have a history as old as that of lattice theory. Distributive lattices have in one sense come full circle in the scheme of things; lattice theory began with Boolean algebras in logic; today there is a great deal of interest in non-Boolean distributive lattices which arise in nonclassical logic.

Along with the general development of lattice theory came the parallel development of the study of distributive lattices. Because of their relative age and the central role they have played, they have received more attention than some other types of lattices, and today distributive lattices are among those best understood. Any distributive lattice is isomorphic to a ring of sets, and any distributive lattice can be considered as a subdirect product of two-element chains. Results like these help to explain the structure of distributive lattices and give one somewhat of an intuitive grasp as to their nature. The analysis of distributive lattices has led to investigations of the distributive parts of other types of lattices and distributive lattices naturally related to arbitrary ones.

The study of distributive lattices has influenced the development of lattice theory in a different way as well. Many of the properties which have been investigated by lattice theorists are weakened forms of distributivity. The obvious example is modularity which in turn has led to the symmetry and semimodularity properties so useful in geometry and combinatorial theory. In another direction the retention of complementation and weakening of distributivity have given rise to some generalizations of Boolean algebras where properties such as orthomodularity have been developed in investigations of functional analysis, mathematical physics and generalized statistics. A thorough knowledge of distributive lattices is clearly indispensable for mathematicians working in lattice theory and related areas.

Any discussion of distributive lattices would be incomplete without mention of universal algebra. Its name goes back at least to Sylvester, and Whitehead in his *Universal algebra* (1898) credits Hamilton and de Morgan with being its founders. These nineteenth century mathematicians recognized and studied universal principles in algebra, but it was not until Birkhoff introduced the modern concept of universal algebra and popularized it in the mid-forties that it began to come into its own. Today it attracts much attention and the works of Cohn, Grätzer, and Pierce have made its ideas readily accessible. Questions arising in universal algebras have led to quite a bit of renewed interest in lattices in general and distributive lattices in particular.

In this setting, it is clear that a book on distributive lattices can occupy an important place in mathematical literature. Balbes and Dwinger have written such a book and it should be well received and widely used. The book is quite carefully put together which is an absolute necessity because of the authors' three-fold approach to distributive lattices—order theoretic, algebraic and categorical. The preliminary sections on universal algebra and category theory are self-contained and enable the reader to progress through the six chapters on general distributive lattices with an appreciation of the new approaches resulting from universal algebra and the relationship of various classes of distributive lattices as best expressed in the language of category theory. In these chapters the main results of distributive lattice theory are gathered together and they provide a foundation for the reading of the last five chapters which deal with special distributive lattices.

The special topics which the authors have chosen include pseudocomplemented distributive lattices; Heyting, Post, de Morgan and Lukasiewicz algebras; and lattices satisfying higher degrees of distributivity. The introductory remarks on the special algebras are particularly helpful in that they include some development from a historical perspective, references for further investigation, and an encapsulation of the role of these special lattices and the results to be emphasized. Balbes and Dwinger point out that they have selected the special topics out of the many possibilities at least partially on the natural bases of their own research interests and taste. However they present references to topics which they have omitted and have wisely selected some topics which are not extensively discussed in other texts. This book differs considerably, for example, from Grätzer's recent text on distributive lattices both in emphasis and approach.

Balbes and Dwinger should have appeal to three audiences. It is a useful reference work for lattice theorists because of the amalgamation of the results in general distributive lattice theory as well as the special topics. In addition the bibliography is quite valuable—not only for source material on subjects in the text but for the references mentioned as suggested for further reading. The mathematician not working in lattice theory can get from this book a good idea of the history and importance of distributive lattices and their role in logic and universal algebra. The book is also suitable as a text for graduate students and the numerous exercises scattered throughout should be quite helpful especially to the novice doing independent reading. In two areas the reviewer wishes things might have been different. The authors use $+$ and $\cdot$ instead of $\vee$ and $\wedge$ throughout; one's preference here is somewhat a matter of "creature comfort" and it must be said that the authors are in good company with respect to their notation (see for example von Neumann's *Continuous geometry*). In a text such as this one, designed to bring the reader to the frontiers of current research, it would have been a natural thing to include in addition to the exercises some specific open problems. Although it is to be hoped that any such collection will soon become out of date, the inclusion of such problems does give a feeling for what the experts are asking and sometimes provides impetus and a challenge, especially for graduate students.

In summary, *Distributive lattices* is worth having. It was carefully planned and well written, providing a survey of the general area, special topics, and information on where to find more. It is a useful reference work for lattice theorists and a good source of information for those not conversant with the field, where perhaps it can kindle a spark of interest in the position and development of one of lattice theory's oldest branches.

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BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 2, March 1976

Pseudo-differential operators, by Michael Taylor, Lecture Notes in Mathematics, No. 416, Springer-Verlag, Berlin, Heidelberg, New York, 1974, iv+155 pp., $7.40.

The reverse of differentiation is integration; the reverse of a linear partial differential operator is, most likely, some kind of integral operator, such as the Newtonian potential. The classical operators of potential theory have been generalized in stages, first to singular integral operators, then to pseudo-differential operators, and on to Fourier integral operators. The heart of these theories is a "*functional calculus*". The singular integral operators, for instance, are mapped homomorphically onto a class of functions, called "symbols" of the operators: The composition of operators corresponds to the pointwise product of symbols, adjoints to complex conjugates, and sums to sums. The homomorphism has a kernel which, in the classical applications, consists of "negligible lower order terms". The