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A GENERALIZED WEIL TYPE REPRESENTATION AND A FUNCTION ANALOGOUS TO e^{-x^2}

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Let $dV(z)$ be the euclidean measure of $\mathbf{C}$, and let $n \geqslant 2$ be a natural number. Put $e(z) = \exp(\pi\sqrt{-1}(z + \bar{z}))$, $\zeta = e(1/n)$, and consider the Hilbert space H_n consisting of all functions Φ on $\mathbf{C}$ such that $\Phi(\zeta z) = \Phi(z)$ and $\|\Phi\| < \infty$, where the norm is coming from the inner product

$$(f_1, f_2) = \int_{\mathbf{C}} f_1(z)\overline{f_2(z)}\, |z|^{2n-4}\, dV(z).$$

Denote by $\Phi \longrightarrow \Phi^*$ the integral linear transformation given by

$$\Phi^*(t) = \int_{\mathbf{C}} \Phi(z)k(zt)\, |z|^{2n-4}\, dV(z)$$

with $k(z) = n^2 \lim_{Y\to\infty} \int_{|z|<Y} e(z^n/w^n)e(w^n)\, dV(z)$.

Denote furthermore by $\sigma = (a, b; c, d)$ an element of $G = SL(2, \mathbf{C})$ for which (a, b) is the first row and (c, d) is the second, and define an operator $r_n(\sigma)$ of H_n for three types of elements $\sigma_1 = (a, 0; 0, a^{-1})$, $\sigma_2 = (1, b; 0, 1)$, and $\sigma_3 = (-c^{-1}, 0; c, 0)$ by $(r_n(\sigma_1)\Phi)(t) = |a|^{n(n-1)/2}\Phi(a^{2/n}t)$, $(r_n(\sigma_2)\Phi)(t) = \Phi(t)e(bt^n)$, and $(r_n(\sigma_3)\Phi)(t) = |c|^{-n(n-1)/2}\Phi^*(c^{-2/n}t)$. Then, it follows from the results, to be announced in [2] in detail, that $r_n(\sigma_i)$ extends multiplicatively to an irreducible unitary representation $\sigma \longrightarrow r_n(\sigma)$ of G of class one on H_n belonging to the supplementary series. If $n = 2$, then $k(z)$ reduces to $e(2z) + e(-2z)$, and $\sigma \longrightarrow r_n(\sigma)$ reduces essentially to a special case of the representation given in [3].

These results, viewed so to speak from the reverse side, yield as a byproduct a representation theoretic characterization of a special function. Namely, we obtain

THEOREM. *Up to a constant factor, the function* $h(t) = tK_{1/n}(2\pi|t|^n)$ *is the only function in* H_n *which is invariant by all* $r_n(\sigma)$ *with* $\sigma \in K = SU(2)$, *where* $K_{1/n}$ *is a modified Bessel function.*

This Theorem follows from the facts, proved in [2], that $h(t)$ is actually invariant by all $r_n(\sigma)$, $(\sigma \in K)$, and that the set of all $r_n(\sigma)h(t)$, $(\sigma \in G)$, is dense in H_n.

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If $n = 2$, then $h(t)$ reduces to $\frac{1}{2}e^{-2\pi|t|^2}$, so that the above theorem with $n = 2$ is easily derived from, and is practically equivalent to, a special case of the result in [1, Chapter 1, §8, Theorem 9], which gives a conceptual characterization of functions of the type e^{-x^2}.

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