Can they be extended to wider contexts than the cases arising in this problem—other graphs, groups; what is their ultimate power?

2. How far can one go toward embedding other classes of graphs or toward results about an arbitrary graph on n vertices? Can one obtain more detailed results such as statements about the existence of embeddings with given index?

3. How do the structures produced here—lists of cycles or flow graphs or whatever—relate to other combinatorial structures—block designs, Latin squares, etc?—is there any relation that allows nontrivial implication in either direction—from or to these results?

4. Is there anything at all in this work that is relevant to integrated circuit problems?

In a similar vein to the given problem are some more difficult questions. The results presented here relate to embeddings of embeddable graphs—when a graph is not embeddable one can raise an analogous question: How much crossing of edges is required to embed the graph? This problem seems more difficult than the original one, because one lacks a characterization of the nature of the "best" embedding; one cannot look for a solution as a triangulation. Very little is known about such "crossing number" problems except conjectured upper bounds on crossing number based upon obvious constructions. Their solution probably awaits a new set of ideas.

Thus, although this volume contains an extremely lucid presentation of a complete solution to the Heawood mapping problem, it should not necessarily be considered the last word on the general subject. Of course this is only one more reason why it deserves to be read.

D. J. KLEITMAN

Differential analysis on complex manifolds, by R. O. Wells, Jr., Prentice-Hall, Englewood Cliffs, N.J., 1973, x+252 pp., \$13.95.

Until the late 1940's it seems that compact complex manifolds were only studied occasionally and even then were not studied as a class of intrinsically interesting objects. In fact most examples of compact complex manifolds were either submanifolds of complex projective space P^n (such manifolds will be called *projective*) or else were Kähler manifolds (such as the nonalgebraic tori). It seems that Hopf's simple construction in 1948 of a non-Kählerian compact complex manifold with $\mathbf{C}^2-\{(0,0)\}$ as universal covering made the study of complex manifolds much more interesting.

There are some obvious and interesting things that one can ask about complex manifolds (these questions, of course, apply to differentiable manifolds also). For example, what relationships hold between the topology of a complex manifold and its complex structure? How are the properties of a complex manifold related to the properties of certain function spaces (e.g. holomorphic functions, strictly plurisubharmonic functions)? How do you distinguish two complex manifolds? What is the relationship between two different complex structures on the same differentiable or topological manifold? Of course these are hard questions, but they are very attractive and one is repeatedly drawn to them.

There have been some recent books which give an introduction to some aspects of the study of complex manifolds. The book of R. O. Wells covers more material than any of these, and also brings the reader closer to the point where he can appreciate the work of Griffiths, Kodaira, and others who are attempting to answer some of the fundamental questions about complex manifolds.

The aim of the book is to introduce the reader to some of the language, techniques, and theorems that are being used in the current literature on complex manifolds. The book is probably most useful to a second (or third) year graduate student who has some knowledge of differential geometry and who knows a bit about such things as Fourier transforms and multilinear algebra.

The first chapter introduces manifolds (real and complex) and vector bundles. This chapter is nicely done and includes a careful discussion of the complex Grassmannians. The complex tangent and cotangent bundles are used as examples of vector bundles, and a discussion of almost complex structures is given. There is also a brief discussion of the universal bundle on a Grassmannian. I found only one misprint in this chapter.

Chapter II gives a 30 page discussion of sheaf theory. This is somewhat brief, but no encyclopedic treatment is needed in a book of this sort. For a more detailed discussion the reader could consult R. Godement, *Topologie algebrique et theorie des faisceaux*, Hermann & Cie, Paris, 1964. The main point of this chapter is to define the cohomology groups of a sheaf and then prove the theorems of de Rham and Dolbeault. These theorems are proved (as usual) by using the lemma of Poincaré (resp. Grothendieck) to produce a fine resolution of the sheaf of locally constant R valued functions (resp. the sheaf of holomorphic p-forms). This, of course, is not the original proof of de Rham's theorem, but it has become the most well-known proof because of its simplicity. It demonstrates very clearly that sheaves summarize an enormous amount of information. This chapter has very few misprints and is well written.

The next chapter gives the definition of Chern classes and discusses some of their functorial properties. Chern classes are defined as certain cohomology classes of real differential forms and via de Rham's theorem are thus real (not integral) cohomology classes. However it is later shown that Chern classes are in the image of the integral cohomology classes under the natural homomorphism. This approach is due to Weil, Chern, and Bott. In order to give the definitions, a certain amount of differential geometry has to be introduced (connections, curvature, Hermitian metrics, etc.). The reader might also like to consult Kobayashi and Nomizu, *Foundations of differential geometry*, Vols. I, II, Wiley, New York, 1963, 1969. There is an explicit discussion of the Chern class of a holomorphic line bundle. This involves looking at the proof of the de Rham theorem. The reviewer has checked this very carefully and believes that the signs are correct! This is not so unimportant since signs play a crucial role in the Kodaira vanishing theorem. It is also true that some sources in the literature have a mistake in the sign.

Chapter IV does the Hodge theory on a compact differential manifold. The author does this from scratch, and there are some defects with the chapter as now written. For instance the reviewer counted 27 misprints in this chapter. The exposition is rather difficult to follow at times and the reader might prefer to consult R. Narasimhan, *Analysis on real and complex manifolds*, North-Holland, Amsterdam, 1968, or F. Warner, *Foundations of differentiable manifolds and Lie groups*, Scott, Foresman, and Company, Glenview Illinois, 1971, for example, on some of this material. The techniques used are from the theory of pseudodifferential operators. This is a rather sophisticated point of view and accounts for part of the problem. The other difficulty is more serious however in that some of the proofs are incorrect as they stand. For example the proof of the semicontinuity theorem, Theorem 4.13, pp. 146–148, is not correct as it stands. It can be corrected, and the correct proof will presumably appear in the next printing. The other mistakes in this chapter will probably also be corrected in the next printing. The (correct) proof of the semicontinuity theorem is very elegant and a worthy addition to the literature. The rest of the chapter is the standard application of the basic results in elliptic operator theory to prove a generalization of the Hodge theorem for elliptic complexes, and then to apply this result to get the representation theorems for certain cohomology groups on compact manifolds as harmonic forms. A crucial consequence is that the groups in question are finite dimensional.

The next chapter begins with a discussion of some algebra on a Hermitian vector space. The reviewer found this material condensed and confusing. In fact this portion of the book was read almost simultaneously with selected sections from A. Weil, *Introduction à l'étude des variétés*

*Kählerienne**s***, Hermann & Cie, Paris, 1958. Read together, these books should be enough to explain the Hermitian algebra. This algebra is applied together with the Hodge theory to prove some of the pretty classical results on Kähler manifolds. For example the Hodge decomposition theorem, decomposing a cohomology class into a sum of harmonic (p, q) forms is proved. The Lefschetz decomposition theorem is also proved, and the Hodge-Riemann bilinear relations are discussed. This is done on the primitive cohomology of a Kähler manifold and an example is produced to show that the result is only valid on the primitive cohomology. Unfortunately there is a mistake in the computation on p. 200 which invalidates the example. There are a fair number of misprints in this chapter, but they generally do not detract from its quality. The reviewer found that after one gets past the algebra the rest is well written and gives an interesting introduction to the papers of Griffiths on periods of integrals on algebraic manifolds.

The discussion of Chapter VI is directed toward a proof of Kodaira's theorem that a Hodge manifold is projective. The proof follows Kodaira's original proof. One first proves Kodaira's vanishing theorem, and then makes an application of this result to the blow up of the Hodge manifold to produce enough sections to give an embedding in complex projective space. The proof of the vanishing theorem differs from Kodaira's in that Nakano's inequality is the crucial ingredient. The reviewer thoroughly enjoyed this chapter and found the exposition to be very clear. There are some confusing misprints in the discussion of the canonical bundle but it is an easy task to correct them. The reader should compare this chapter with the last few pages of the book by Gunning and Rossi, *Analytic functions of several variables*, Prentice-Hall, Englewood Cliffs, N.J., 1965, where a discussion of Grauert's proof of this theorem is given.

The topics treated in the book under review are fundamental. Every complex analyst should know (or learn) this basic material, and Wells' book is a good reference for these essential results about complex manifolds.

JAMES A. MORROW

Topics in analytic number theory, by Hans Rademacher, Die Grundlehren der math. Wissenschaften, Band 169, Springer-Verlag, Berlin, 1973, ix+320 pp.

Topics in analytic number theory by Hans Rademacher covers all the classical aspects of a subject which is presently undergoing a revolution. According to the editors, Professor Rademacher had been working on this