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SOME INTEGRAL INEQUALITIES OF TWO GEOMETRIC INVARIANTS

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Let M be an n-dimensional manifold immersed in a euclidean m-space E^m. Let S and α be the length of second fundamental form and the length of mean curvature vector and let ρ be the scalar curvature of M. Then $\rho = n^2\alpha^2 - S^2$. From Proposition 2.2 of [2], ρ satisfies the following pinching property:

$$-S^2 \leqslant \rho \leqslant (n-1)S^2.$$

Let F be a field and let $H_i(M; F)$ be the ith homology group of M over F. We define a topological invariant $\beta(M)$ by

$$\beta(M) = \max\left\{\sum_{i=0}^{n} \dim H_i(M; F)\colon F \text{ fields}\right\}.$$

The purpose of this note is to announce the following results. The detailed proofs will appear elsewhere.

THEOREM 1. *Let M be an n-dimensional closed manifold immersed in a euclidean m-space E^m. Then*

$$\int_M S^n dV \geqslant \gamma\beta(M), \tag{1}$$

where

$$\gamma = \begin{cases} n^{n/2}c_n/2, & \text{if } n \text{ is even}, \\ n^{(n+1)/2}c_{n+1}c_{m-1}/2c_m(m-1)^{1/2}, & \text{if } n \text{ is odd}, \end{cases} \tag{2}$$

and c_n is the area of unit n-spheres. The equality sign holds only when M is imbedded as a hypersphere of a linear $(n + 1)$-subspace of E^m.

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This theorem was proved by using several lemmas given in [1, Chapter VII]. By applying Theorem 1 we have the following theorem on total mean curvature.

THEOREM 2. *Let* M *be an n-dimensional closed manifold immersed in* E^m. *If there is a* $\delta > -1$ *such that* $\delta S^2 \leqslant \rho \leqslant (n-1)S^2$, *then*

$$\int_M \alpha^n dV \geqslant (1+\delta)^{n/2}\gamma\beta(M)/n^n. \tag{3}$$

The equality sign of (3) *holds only when* $\delta = n-1$ *and* M *is imbedded as a hypersphere in an* $(n+1)$*-dimensional linear subspace of* E^m.

Some special cases of these results were obtained in [2].

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