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MINIMUM COVERS FOR ARCS OF CONSTANT LENGTH

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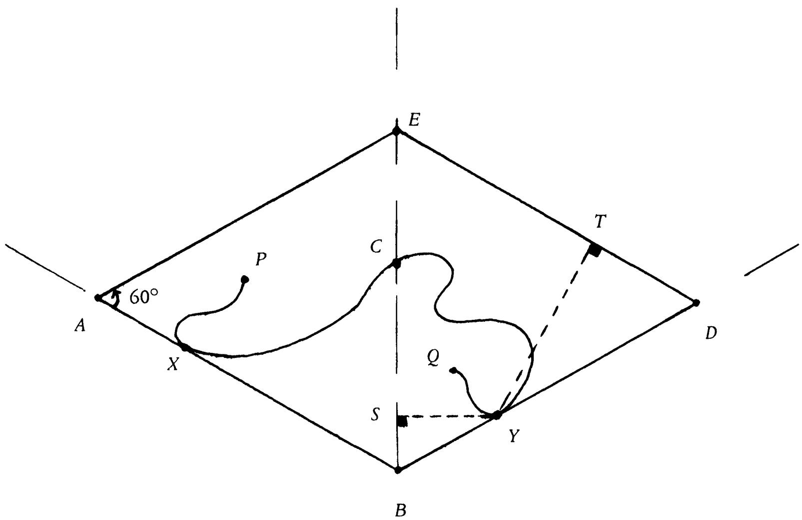
Recently Gerriets [**1**] showed that a certain convex closed region with area less than $0.3214L^2$ covers any arc of length L. This is an improvement to Wetzel's results [**3**] on the famous and elusive "Worm Problem" of Leo Moser [**2**]: What is the (convex) region of smallest area which will accommodate every arc of length L? Wetzel showed that a certain truncated sector with area less than $0.34423L^2$ covers all arcs of length L. By slightly modifying the region considered by Gerriets, we obtain a region with area less than $0.2887L^2$ which covers any arc of length L.

THEOREM. *The closed region whose boundary is a rhombus with major diagonal L and minor diagonal $L/3^{1/2}$ covers any arc of length L.*

Herein we give a sketch of the proof. Details and other results will appear elsewhere. Let PQ denote an arc of length L and with center C whose two subarcs are PC and CQ. "Slide" the arc PQ along BE toward B so that C is always incident with BE and PQ becomes "tangent" to AB or BD (see the figure below) at the points X or Y. It is possible that all such



orientations of PQ by rotation allow only one of the arcs PC or CQ to be

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tangent to the angle ABD with all other points of the arc PQ lying on or above the angle. Assume for the present, however, that there are two tangent points X, Y on PC, CQ which lie on the segments AB and BD, respectively. Construct the segments SY and YT perpendicular to BE and DE, respectively (the case when a point between C and Y meets DE is handled in a similar way to the case under discussion). If CQ agrees with the segments SY and YT, then the length of CQ is exactly $L/2$ and, hence, is covered by the region R described in the Theorem. If CQ does not agree with SY and YT, then in order for CQ to get to the boundary, its length must exceed the length of SYT (which is $L/2$), an impossibility. So CQ is covered by R and, similarly, PC is also covered. Symmetry of R dispenses with the case that PQ has only one subarc tangent to ABD.

It can be shown that the region R can be truncated to obtain a region with area less than $0.2861L^2$ which covers any arc of length L.

The authors wish to thank Professor John E. Wetzel for sharing the results in [3] prior to publication.

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