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COMPACTIFICATIONS OF C^2

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1. **Introduction.** By a *compactification* of C^2 we mean a nonsingular compact complex manifold M of complex dimension 2 which contains a nonempty nowhere dense closed analytic subset A such that $M - A$ is biholomorphic to C^2. It is not hard to verify that A is a compact connected one-dimensional analytic set, hence a finite union of irreducible curves. By blowing up certain points of A we may assume that A has the following properties.

(1) $A = \bigcup_{i=1}^{k} \Gamma_i$, where Γ_i is a nonsingular connected algebraic curve.

(2) Γ_i intersects Γ_j normally (if at all).

(3) $\Gamma_i \cap \Gamma_j \cap \Gamma_k = \varnothing$ for any three distinct indices.

(4) If the self-intersection $(\Gamma_i)^2 = -1$, then Γ_i meets at least three other curves Γ_j.

We call such a compactification a *minimal normal compactification* of C^2. The purpose of this note is to announce a list of all minimal normal compactifications of C^2. The proofs will appear elsewhere.

2. **Sketch of method.** The construction and proofs rely heavily on [3] and [4]. It is not hard to prove (see [5]) that each $\Gamma_i \cong P^1(C)$. A theorem of van de Ven [4] says that M is necessarily algebraic. A result of Ramanujam [3] says that the graph of A is linear. One then uses a surgical technique to find what possible selfintersection numbers the Γ_i can have. One step in the proof uses a theorem of Mumford [2] to compute the fundamental group of the boundary of a tubular neighborhood of A. One can then produce a list of possible graphs. One can prove that the compactifications corresponding to these graphs actually occur and are uniquely determined by the graphs. This has as a corollary the fact that all compactifications of C^2 are rational, a result conjectured by van de Ven and recently proved by Kodaira [1] by different techniques.

3. **The list of graphs.** The notation is as follows. Each line represents a point of intersection and each circle ("vertex") represents a nonsingular rational curve ($P^1(C)$). The number adjacent to each circle is the self-

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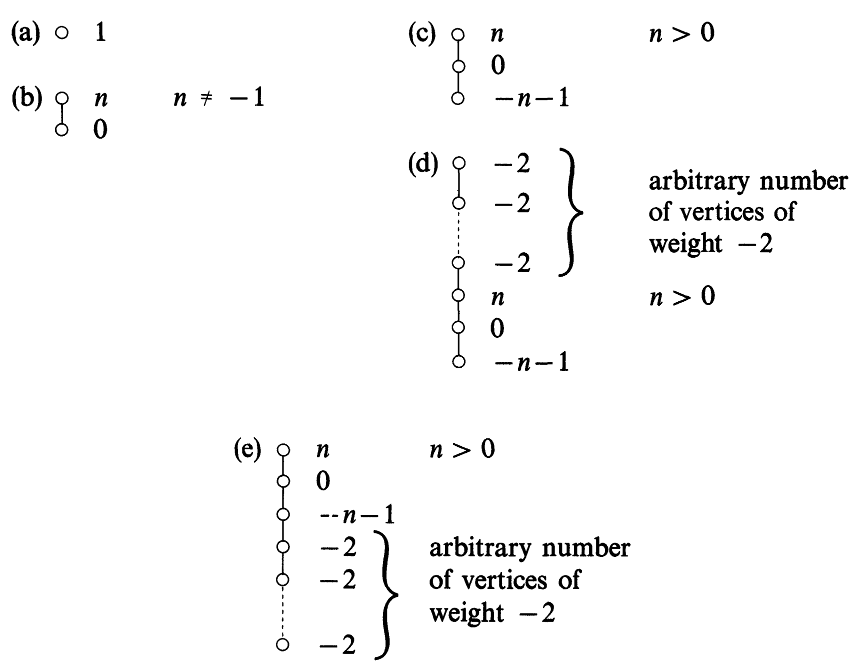
intersection of the rational curve. The graphs of compactifications are shown on the following two pages:

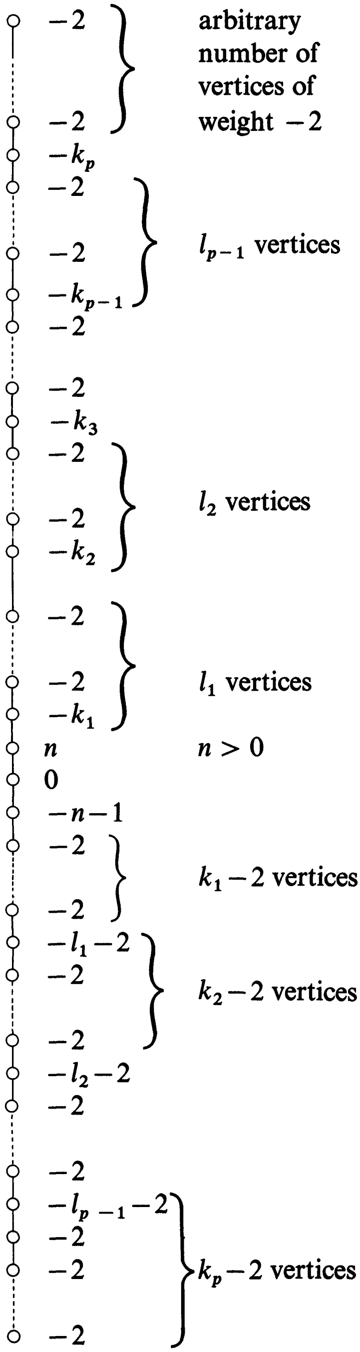
In graph (f) one has $k_i \geqq 2$, $l_i \geqq 0$. If $k_1 = 2$, then $l_1 \geqq 1$ and there are no $(k_2 - 2 = 0)$ vertices with weight -2 adjacent to the vertex with weight $-n-1$. We assume $k_i > 2$ for $i > 1$. Thus $l_1 - 1$ is the number of vertices with weight -2 between the vertex with weight $-k_1$ and the first vertex with weight $-k_2 < -2$. If there is no such vertex $l_1 - 1$ is the number of vertices with weight -2 from the vertex with weight $-k_1$ to the end. If there are no vertices beyond $-k_1$ we set $l_1 = 1$. If there are no vertices beyond $-n-1$, then we are in case (d).

Below the vertex with weight $-n-1$ there are, first, $k_1 - 2$ vertices with weight -2 and then appears a vertex with weight $-l_1 - 2$. In this picture we assume there are some vertices above the vertex with weight n. We also assume $k_j > 2$ unless $j = 1$ in which case we allow $k_j = k_1 = 2$.

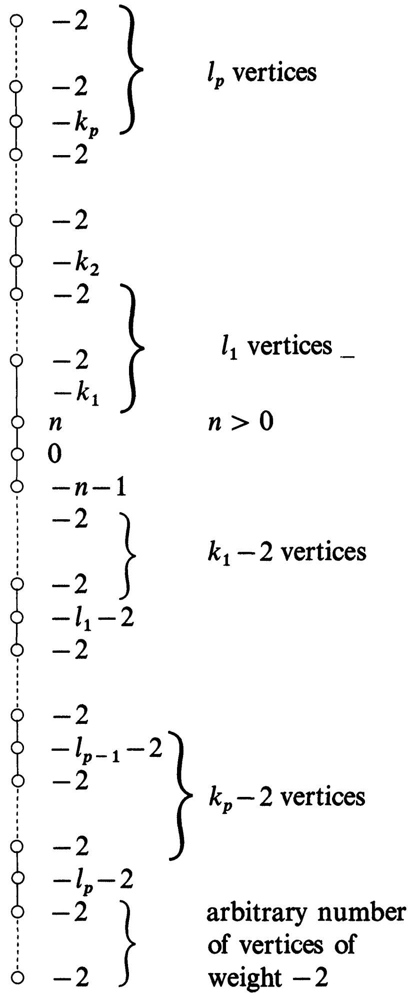
The graph (g) is essentially the same as that of case (f). The essential difference is that we allow the case with no vertices above the one with weight n, and hence get (e) as a special case.



(f)


−2
−2
arbitrary number of vertices of weight −2
−k_p
−2
−2
−k_{p−1}
l_{p−1} vertices
−2
−2
−k_3
−2
−2
−k_2
l_2 vertices
−2
−2
−k_1
l_1 vertices
n
n > 0
0
−n−1
−2
−2
k_1−2 vertices
−l_1−2
−2
−2
k_2−2 vertices
−l_2−2
−2
−2
−l_p −1−2
−2
−2
−2
k_p−2 vertices

(g)


−2
−2
−k_p
l_p vertices
−2
−2
−k_2
−2
−2
−k_1
l_1 vertices
n
n > 0
0
−n−1
−2
−2
k_1−2 vertices
−l_1−2
−2
−2
−l_{p−1}−2
−2
−2
k_p−2 vertices
−l_p−2
−2
−2
arbitrary number of vertices of weight −2

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