Nothing even remotely similar to the negative laws of thermodynamics has been discovered which would forbid the existence of a large but simply describable mechanism, perhaps realizable within the means of present or future technology, which would be as intelligent as an animal or man.}

The book is very well written. Its clear compact and very engaging style and its large number of ideas and open problems make it perfect material for study in seminars, and it should have a strong influence on future writers in this subject. It is also the first purely mathematical monograph about certain aspects of learning and perception, and this subject may become the most important theory of 20th century mathematics.

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Methods of Hilbert Spaces, by Krzysztof Maurin. PWN-Polish Scientific Publishers, 1967. Vol. 45 in the series Monografie Matematyczne of Polska Akademia Nauk. 554 pp. and *General Eigenfunction Expansions and Unitary Representations of Topological Groups*, by Krzysztof Maurin. PWN-Polish Scientific Publishers, 1968. Vol. 48 in the series Monografie Matematyczne of Polska Akademia Nauk. 367 pp.

Each of these books has a strong allure for the modern analyst, for Maurin has assembled unique collections of interesting topics. *Methods of Hilbert spaces* (hereafter MHS) begins with nine chapters which provide an introduction to the abstract theory, including the basic definitions and geometry of Hilbert spaces and locally convex spaces, the spectral theory of selfadjoint, unitary and compact operators, and the general spectral theory of commutative C^*-algebras via the Gelfand theory for commutative Banach algebras. In the next fourteen chapters this general theory is applied to one parameter semigroups, elliptic partial differential equations, the orthogonal projection method, Bochner's theorem on the existence of euclidean embeddings of compact analytic Riemannian manifolds, a general theory of eigenfunction expansions, a generalized Fourier separation of variables method, the Hodge-Kodaira theory of harmonic fields (forms) on Riemannian manifolds, the early theorems of Hörmander on partial differential operators of principal type, ergodic theory, almost periodic functions, representations of topological groups, and excerpts from Hörmander's presentation of several complex variable theory including the Cousin and Levi problems. Part I of *General eigenfunction expansions and unitary representations of topological groups* (hereafter GEE) provides a review and extension of the material in MHS on the general spectral theorem of von Neumann, the direct integral decomposition of von Neumann algebras and the basic facts about nuclear spaces and then applies these to a general theory of eigenfunction expansions. Part II is devoted to unitary representation theory including the Peter-Weyl theory for compact groups, the Mautner and von Neumann direct integral decomposition theorems for representations of locally compact groups, Harish-Chandra's theorem that semisimple Lie groups are type I, some abstract character theory including the Pontryagin duality theorem, an introduction to the theory of spherical and automorphic functions and finally the Mackey theory of induced representations and its applications to quantum theory, ergodic theory and the representation theory for nilpotent Lie groups.

The reader, with diligent work, can attain an understanding of this material, but his achievement will be more a tribute to the reader and the mathematicians whose work is considered than to the author. It is unfortunate that Maurin's taste for some of the most interesting areas of modern analysis does not carry over to a taste for the careful presentation such work deserves. He is inattentive to the need for precision in the presentation of arguments, to consistency of notation, and to the technical details which make for a useful reference work.

Let us examine each of the books more closely.

MHS is intended as an introductory work. As Maurin says "on the face of it, (this book) does not presuppose any previous knowledge on the part of the reader except the knowledge of the elements of the integral calculus in many variables, general topology and the theory of Lebesgue integration" (from the introduction to the Polish edition of 1959, reprinted in the present expanded English version). But, because of the amazingly wide range of topics considered and because of the nature of Maurin's presentation, one actually must have a reasonably extensive exposure to many of the topics, not only to appreciate the results obtained, but even to understand and follow their presentation.

The treatment of the theory of partial differential equations illustrates many of the difficulties of Maurin's presentation. Certainly some of the most impressive uses of Hilbert space methods and functional analytic techniques have been in the solution of important problems of this theory, and Maurin rightfully makes these topics the central point of MHS. The first problem is that he is careless in fixing the notational foundations in these sections. Three different definitions of the partial derivative symbol D^α lead to repeated ambiguity as to what the formal adjoint of a partial differential operator is, as well as to confusion over the behaviour of constant coefficient operators under the Fourier transform. And three distinct definitions of the Sobolev type spaces H^m, two of which occur within two pages of each other (pp. 410, 412), sometimes make it difficult to know exactly what a theorem is asserting.

More seriously, the presentation of the basic results concerning partial differential equations seems poorly organized and executed. Often there is no clear statement of the problem being solved. The discussion of the central problem of the regularity of weak solutions of elliptic equations fails to make clear the relationship between Hilbert space techniques and the more classical analytic results which are used. For instance, local regularity of weak solutions is deduced from an unproved (but referenced) result which states that an elliptic system has a local fundamental solution given by integration against a kernel function with certain regularity properties. Thus the crux of the problem is entirely removed from the domain of Hilbert space methods. In later discussions of the Dirichlet boundary problem the question of existence and of global regularity are not sufficiently separated. As a result, one finds most of the standard tools, Gårding's inequality, the Sobolev and Rellich lemmas, etc., but their full value never seems to get exploited. Thus there is no result which states that for a C^∞ elliptic operator A a weak solution of $Au = f$ is C^∞ when f is C^∞. In view of Maurin's use of hypoellipticity hypotheses in his chapters on eigenfunction expansions this is a surprising omission.

Other examples of Maurin's carelessness occur in simple material and undermine one's confidence in the treatment of more advanced topics. For instance, in the second chapter, after having shown that a continuous linear map, $A: X \to X_1$, between *normed* spaces induces a continuous 1-1 map of $X/\mathrm{Ker}(A)$ into X_1, Maurin claims to have proved the (false) result that if A maps onto X_1 then $X/\mathrm{Ker}(A)$ and X_1 are isomorphic as normed spaces. Here no hypothesis of completeness is stated, although in Chapter 3 a correct version of this isomorphism theorem is proved for Fréchet spaces. Another example of the confusion in the early material occurs in the discussion of the Gelfand theory of the maximal ideal space for a commutative Banach algebra. Maurin overlooks the crucial point that the spectrum of an element depends on which algebra containing it one considers, and thus states a theorem (p. 175) which has two interpretations, one of which is false.

Finally, there are the technical problems. There are far too many proof reading errors—missing asterisks on adjoint operators, missing complex conjugate or closure signs, interchanges of the letters u and v, and incorrect interchapter cross-references. (Undoubtedly this last is a result of the careless checking of the new references introduced by the revision of the Polish edition.) One of the more curious errors occurs at p. 229 where several pages of material have been omitted, leading to an incomprehensible transition in the text. (The missing material can be found in the exposition of the same topic in GEE, pp. 182–183.)

GEE has more realistic aims (i.e., it admits to being an advanced monograph) and exhibits more attention to matters of organization and presentation than MHS. It provides an interesting survey of the topics in its title.

One reason for describing GEE as a survey is that many of the more interesting and difficult results are either briefly sketched or else quoted from other sources without proof. There is nothing wrong with this approach, but the reader should be aware of what to expect. For instance, it would require a more complete study of the direct integral decomposition theory for von Neumann algebras to fill in the details of the proofs of the von Neumann and Mautner theorems on p. 163. Similarly, the presentation of Godement's proof that semisimple Lie groups are type I depends on more knowledge about the connection between finite traces and factors of type I than is developed in GEE. Examples of dependence on quoted but unproved results are furnished by all of Mackey's theorems on disjointness and quasi-equivalence of factor representations, as well as his results relating multiplicity free representations to measure classes in the space $\hat{G}$ of equivalence classes of irreducible representations. (It should be noted that although the Borel structure on $\hat{G}$ is referred to, it is never defined.) Mackey's theorems on induced representations are also simply quoted, although a partial proof of the imprimitivity theorem is given.

A unique and interesting element in the presentation of GEE is the repeated occurrence of what Maurin calls Gelfand triplets. A Gelfand triplet, $\Phi \subset H \subset \Phi'$, consists of a Hilbert space H together with a dense nuclear subspace Φ and its dual Φ'. The basic usefulness of such a triplet occurs in the nuclear variant of the complete spectral theorem due to Gelfand. Here a strongly commuting family of operators $\mathscr{A}$ is used to produce a direct integral decomposition of H which diagonalizes $\mathscr{A}$. Then if Φ is $\mathscr{A}$-invariant, the nuclearity of Φ allows one to choose a determining subset of Φ' consisting of eigenvectors for the transpose of the family $\mathscr{A}$. This recovers much of the eigenvector analysis of the classical finite-dimensional spectral theorem. Throughout both parts of GEE an attempt is made to identify Gelfand triplets and potential settings for the nuclear spectral theorem.

The problems with this book are similar to those of MHS. Maurin's carelessness recurs here. At times it takes the form of sloppy notation. An example of this is in the discussion of the Peter-Weyl theory for a compact group G where on one page H^α stands for the $n(\alpha)$-dimensional space of an irreducible representation α of G, while a page later it stands for the subspace of $L^2(G)$ of dimension $n(\alpha)^2$ which is spanned by the matrix functions of α. This ambiguity in the meaning of H^α may lie behind the fact that all the formulae on p. 169 for the expansion of a central function on G in terms of irreducible characters are off by a factor of $n(\alpha)$. At other times the lack of care leads to more serious errors. An example is furnished by the proof of the false (but luckily unused) result (p. 274) that if G is unimodular and K is a closed subgroup then there exists a G-invariant measure on G/K where one finds the reason "... for a subgroup of a unimodular group is unimodular." This occurs despite the example on p. 132 which computes the Haar measures of the group of upper triangular matrices and which is followed by the warning "*This example shows that a subgroup of a unimodular group need not be unimodular.*" (Emphasis in the original.) A second example appears in the section on duality for locally compact abelian groups where a lemma is "proved" which claims that the Fourier transform takes continuous functions of compact support on G into a dense subset of L^1 of the dual group. (Although the "proof" gives no information as to what was actually intended, it seems likely that what should have been here is the corresponding result for the behaviour of the span of the continuous positive definite functions of compact support under the Fourier transform.)

Maurin also occasionally omits detailed references. Thus in the midst of the discussion on pp. 279–280 of Mackey's techniques for analyzing the irreducible representations of a semidirect product, Maurin gives a half page quote attributed to Mackey followed by the statement of numerous unproved results, *all without references.* (Niels Poulsen identified the source of the quote as Mackey's appendix to I. E. Segal's *Mathematical problems of relativistic physics*, Amer. Math. Soc., 1963. This book does not appear in Maurin's bibliography.)

One final comment. Maurin's presentation of the Pontryagin duality theorem closely resembles that given in the books by W. Rudin and G. Bachman. Unfortunately each proof uses spurious reasoning to show that *the image, $\alpha(G)$, of G in its second dual is closed.* Each claims to deduce this fact from the previously established result that $\alpha(G)$ is locally compact in the topology induced from the second dual *without using the fact that $\alpha(G)$ is a subgroup.* Such reasoning cannot work as the example of $(0, 1)$ in $[0, 1]$ shows. A correct version of this proof (perhaps the original) was given by Cartan and Godement (Ann. Sci. École Norm. Sup, 1947) and uses the facts that (1) a locally compact subspace is locally closed, and (2) a locally closed *subgroup* of a topological group is actually closed. Hopefully the proliferation of spurious proofs of the duality theorem can be brought to a halt.

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