RESEARCH PROBLEMS

10. George U. Brauer. *Sets of convergence of exponential series.*

Let $\{\alpha_n\}$ denote a sequence of real numbers which is contained in the interval $[0, 2\pi]$ and dense in a subinterval of length $\pi/2$. If E is a given subset of the set of natural numbers, does there exist a series of the form

$$\sum_{n=0}^{\infty} c_n \exp(ik\alpha_n)$$

which converges when the natural number k is in E and diverges when the natural number k is not in E? (Received December 10, 1965.)

11. Richard Bellman. *Truncation of Infinite System of Ordinary Differential Equations*

Consider the linear partial differential equation $u_t = k(x)u_{xx}$, $u(x, 0) = g(x)$, $u(0, t) = u(1, t) = 0$, $t > 0$. Let $u(x, t) \sim \sum_{n=1}^{\infty} u_n(t) \sin n\pi x$, $g(x) \sim \sum_{n=1}^{\infty} g_n \sin n\pi x$, and let $u_n'(t) = \sum_{m=1}^{\infty} a_{nm}u_m(t)$, $u_n(0) = g_n$, $n = 1, 2, \cdots$, denote the infinite system of linear differential equations obtained from the partial differential equation.

Consider the truncated system

$$\frac{du_n^{(N)}}{dt} = \sum_{m=1}^{N} a_{nm}u_m^{(N)}(t), \quad u_n^{(N)}(0) = g_n, \quad n = 1, 2, \cdots, N.$$

Assume that $k(x) \geqq k_0 > 0$ for $0 \leqq x \leqq 1$. What are the weakest additional conditions that must be imposed upon $k(x)$ and upon $g(x)$ to ensure that $\lim u_n^{(N)}$ exists as $N \to \infty$, and that it converges to the nth Fourier coefficient of the solution of the partial differential equation? (Received December 10, 1965.)

12. J. O. C. Ezeilo. *Periodic solutions of differential equations.*

Consider the real differential equation

$$\dddot{x} + a\ddot{x} + \dot{x} + a \sin x = 0 \tag{1}$$

in which a is a constant. For sufficiently small x, the equation (1) takes the approximate form

$$\dddot{x} + a\ddot{x} + \dot{x} + ax = 0$$

which has periodic solutions of period 2π. It is thus reasonable to anticipate that the original equation (1) itself does have nontrivial periodic solutions for arbitrary values of a. Can this be proved? (Received December 31, 1965.)