RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable.

CONTRACTIBILITY OF CERTAIN SEMIGROUPS

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1. It has long been known that a compact, connected, triangulable topological group, G, must have its Euler-Poincaré number $\nu(G)=0$. The analogous result is not true if G is replaced by S, a topological semigroup with identity. (Witness the unit interval under multiplication.) However, we will show: If S is a compact, connected, triangulable semigroup with identity, then $\nu(S)=0$ or $\nu(S)=1$ and, in the latter case, S is contractible.

2. We follow, in part, the terminology of [1].

Let S be any topological semigroup, then R denotes a minimal right ideal of S, if such exists.

Let us say that a semigroup S satisfies $*$ if $x\in S$ implies there exists a $y\in S$ such that $xy=y$.

We recall that a space is contractible if the identity mapping is homotopic to a constant map.

LEMMA. *Let S be a compact, arcwise connected topological semigroup with identity element e. If S satisfies $*$, then S is contractible.*

PROOF. It is known [1] that S has a minimal right ideal R and that $a\in R$ implies $aR=R=aS$. Fix any $x\in R$, then by $*$ above, there is a $y\in S$ such that $y=xy\in xS=R$. If $z\in R$, then there is a $z'\in R$ such that $yz'=z$; thus $xz=xyz'=yz'=z$. It follows that $xy=y$ for any $x, y\in R$.

Let $i\colon S\to S$ be the identity mapping and let $x\in R$. Since S is arcwise connected, there is an arc from e to x; let $p\colon I\to S$ be such an arc (I denotes the unit interval), with $p(0)=e$ and $p(1)=x$.

Define

$$h\colon S\times I\to S$$

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by $h(s, t) = p(t)sp(t)$. Then $h(s, 0) = s = i(s)$ and $h(s, 1) = (xs)x = x$. Hence i is homotopic to a constant map and thus S is contractible.

REMARKS. It should be observed that in the proof above the arcwise connectedness of S can be replaced by the existence of an arc between e and some element of the minimal ideal of S.

The necessity of a two-sided identity is clearly indicated by the example of two tangent circles with multiplication $xy = y$.

THEOREM. *Let S be a compact, arcwise connected, topological semigroup with identity e. If S has no fixed point free deformations, then S is contractible.*

PROOF. It is easy to see that multiplication by a fixed element of S is homotopic to the identity, thus it must have a fixed point and, hence, S satisfies $*$. It follows from the lemma that S is contractible.

COROLLARY. *Let S be compact and triangulable with nonzero Euler-Poincaré characteristic. If S is a topological semigroup with an identity then S is contractible.*

It need only be noted that S has no fixed point free deformations by a well-known corollary to the Lefschetz fixed-point theorem [2].

REFERENCES

1. A. D. Wallace, *The structure of topological semigroups*, Bull. Amer. Math. Soc. **61** (1955), 95–112.

2. D. G. Bourgin, *Modern algebraic topology*, Macmillan, New York, 1963.

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