RESEARCH PROBLEMS

9. George Brauer: *Sets of divergence of exponential series.*

Erdös, Herzog and Piranian show [2], [3] that if E is the union of a set of type F_σ of logarithmic measure zero and a set of type G_δ on the unit circle C, then there exists a Taylor series which diverges on E and converges on $C-E$. In [1] it is shown how to extend these results to ordinary Dirichlet series.

Let $\{\lambda_n\}$ denote a sequence of real numbers tending to infinity such that the second difference $\lambda_{n+1}-2\lambda_n+\lambda_{n-1}$ also tends to infinity; characterize the subsets E of $(-\infty, \infty)$ such that there exists a series $\sum_{n=1}^{\infty} a_n \exp(-i\lambda_n\tau)$ which diverges on E and converges on the complement of E. For example, is it true that if the set of divergence is neither empty nor the entire interval $(-\infty, \infty)$, then both the set of divergence and the set of convergence are dense in $(-\infty, \infty)$?

References

1. G. Brauer, *Sets of convergence of ordinary Dirichlet series*, Duke Math. J. **21** (1954), 593–594.

2. F. Herzog and G. Piranian, *Sets of convergence of Taylor series*. I, Duke Math J. **16** (1949), 529–534.

3. P. Erdös, F. Herzog and G. Piranian, *Sets of divergence of Taylor series and of trigonometric series*, Math. Scand. **2** (1954), 262–266.

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