BOOK REVIEWS

Cohomology operations. Lectures by N. E. Steenrod. Written and revised by D. B. A. Epstein. Annals of Mathematics Studies, No. 50. Princeton University Press, Princeton, N. J., 1962. 139 pp. $3.00.

Cohomology operations have had, since their discovery in the 1940's, many important applications in algebraic and differential topology. In spite of this, there has been essentially no source outside of the original papers for the study of these operations. The book under review here represents a major step towards filling this gap. It treats the Steenrod reduced powers, giving a new improved method of construction (which does not appear in print elsewhere) together with many of their interesting applications.

The book begins with a set of axioms for the Steenrod algebra $\mathcal{A}(2)$, postponing the existence and uniqueness to the final chapters. The vector space basis of Adem ("Relations on Steenrod Powers of Cohomology Classes," *Algebraic geometry and topology*, Princeton, 1957) and Cartan (*Sur l'iteration des operations de Steenrod*, Comment. Math. Helv. **29** (1955), 40–58) is obtained and it is shown that the indecomposable elements of $\mathcal{A}(2)$ are of the form $S_q^{2^i}$. This second fact is shown to put severe restrictions on the kinds of truncated polynomial rings that can occur as the mod 2 cohomology ring of a space and also to show that, if $f\colon S^{2n-1}\to S^n$ has odd Hopf invariant, then n is a power of 2. It is also shown that, for n even, maps $S^{2n-1}\to S^n$ exist with any even Hopf invariant.

In Chapter 2, the structure of $\mathcal{A}(2)$ as a Hopf algebra is studied and the results of Milnor (*The Steenrod algebra and its dual*, Ann. of Math. (2) **67** (1958), 150–171) on the dual Hopf algebra $\mathcal{A}(2)^*$ are obtained. In Chapter 3, the nonembedding theorems of Thom and Hopf are proved, Thom's theorem dealing with the embeddability of a compact space in a sphere and Hopf's theorem with the embeddability of an $(n-1)$-manifold in an n-sphere.

Chapter 4 is devoted to the determination of the cohomology rings of the classical groups and the Stiefel manifolds. This is done by exhibiting an explicit cellular structure for these spaces. The action of $\mathcal{A}(2)$ on the mod 2 cohomology of the real Stiefel manifolds is derived and the results applied to obtain an upper bound for the number of linearly independent tangent vector fields on a sphere.

The next chapter develops some of the technical machinery needed in the construction of the reduced powers. The notion of equivariant cohomology is introduced and the cohomology of a group is defined.

Computations are made for the cyclic groups and some results about the transfer homomorphism obtained.

Chapter 6 deals with the reduced pth powers for odd primes p. Results analogous to those of the first two chapters are proved for the Steenrod algebra $\mathcal{A}(p)$ and applications given, including the result that the p-primary component of $\pi_i(S^3)$ is zero for $i<2p$ and Z_p for $i=2p$.

The next two chapters are devoted to the construction of the reduced powers, the verification of the axioms and the uniqueness theorem. Briefly, the construction is as follows. Let K be a complex, π a subgroup of the symmetric group $\mathcal{S}(n)$ on n letters, and W a π-free acyclic complex. If $K^n=K\times K\times \cdots \times K$ (n factors), π acts on $W\times K^n$ by the diagonal action and we denote the quotient by $W\times_\pi K^n$. Now, if u is a cohomology class on K and $u^n=u\times \cdots \times u$ the n-fold external product of u on K^n, we can, under suitable circumstances, extend u^n in a natural way to a class Pu on $W\times_\pi K^n$. Letting d: $K\to K^n$ be the diagonal map, we have a map $1\times_\pi d$: $W\times_\pi K = W/\pi\times K\to W\times_\pi K^n$ and an induced map $(1\times_\pi d)^*$: $H^*(W/\pi\times K)\to H^*(W\times_\pi K^n)$. If the coefficient domain is a field, $H^*(W/\pi\times K)\approx H^*(W/\pi)\otimes H^*(K)$ and we define the set of reduced powers of u to be the elements u_i of $H^*(K)$ where $(1\times_\pi d)^*Pu=\sum v_i\otimes u_i \in H^*(W/\pi)\otimes H^*(K)$. In particular, the Steenrod reduced pth powers are obtained when π is the cyclic group of $\mathcal{S}(p)$ generated by the unique p-cycle.

The concluding chapter of the book is an appendix by D. B. A. Epstein in which purely algebraic derivations are given for some properties of the Steenrod algebra which previously had mixed geometric-algebraic derivations.

The book is extremely well written and the choice of material excellent. The prerequisites have been kept at a minimum so that only a first course in algebraic topology is required. It certainly would be a valuable addition to the library of any topologist and, in fact, to any library already having the introductory books in algebraic topology.

R. H. Szczarba

Differential forms with applications to the physical sciences. (Mathematics in science and engineering, vol. 11, ed. by R. Bellman.) By Harley Flanders. Academic Press, New York, 1963. 12+203 pp. \$7.50.

This is a remarkable book, presenting the fundamental ideas of the geometry of manifolds in a robust, unpedantic and clear manner; in