BOOK REVIEWS

Homology. By Saunders MacLane. (Die Grundlehren der Mathematischen Wissenschaften, Bd. 114.) Springer, Berlin, 1963; Academic Press, New York, 1963. 422 pp. illus. $15.50.

Approximately seven years have elapsed between the appearance of the first book on homological algebra by Cartan and Eilenberg, and the publication of this book on homology by MacLane. Since both books deal with essentially the same material, it is perhaps inevitable that in reviewing the latter we begin by making some comments contrasting it with the former.

The first book was concerned with the presentation of a new subject—Homological Algebra. At the time of its preparation (over ten years ago) it was not clear just what directions this new subject would move in, and its major triumph and delight was the discovery that by replacing free modules by projectives one could define derived functors of Hom and Tensor product to obtain Ext and Tor, and thereby subsume the existing homology theories of algebraic structures under one unifying notion. Therefore the authors devoted themselves to a painstaking exposition of projective and injective resolutions, derived functors, satellites, homological dimension, products, and spectral sequences, and joyfully indicated the connections between these new phenomena and old ones such as the homology of groups, algebras and Lie algebras, the Hilbert theorem on syzygies, and the Künneth formula.

Since that time, the methods of homological algebra have been applied extensively in many areas, and homological techniques have been refined, elaborated, and generalized. Moreover, the value of homological tools and of the functorial approach has been conceded by more than a dozen mathematicians. Thus MacLane approaches his subject not as a magician with a set of tricks, but as an expositor who intends to demonstrate how a viable coherent theory illuminates meaningful questions. His general pattern, then, is to go from the particular to the general, throwing as much light as possible along the way.

It must of course be borne in mind that the lapse of time and development of the subject are not solely responsible for the clarity of presentation of much of the material in this volume. The author evidently believes in concrete examples and concrete applications of general machinery and within the limited scope of the book, does a fine job in providing them. Thus, when he discusses differential groups, he gives several simple topological examples. His discussion of the cohomology of groups includes the topological interpretation of the cohomology groups of a group π as the cohomology of an aspherical space with fundamental group π. His discussion of dimension includes the study of syzygies and separable algebras, and the Wedderburn principal theorem for algebras is included in his chapter on the cohomology of algebraic systems. When treating spectral sequences, he constructs the spectral sequence of a covering and of a group extension, and also obtains the Gysin sequence.

The following brief summary of the contents of the book will give an indication of how some of the more recent applications and developments of homology theory have been incorporated into this volume. The first chapter, Modules, Diagrams, and Functors, is preliminary. Chapter II, Homology of Complexes, contains the basic definitions of complexes, the exact homology sequence, etc. It also includes a section on singular homology. In Chapter III, Extensions and Resolutions, the definition of Ext in terms of long exact sequences (à la Yoneda) is given, and is motivated by considering obstructions to extending homomorphisms. Injective envelopes are also included in this chapter. Chapter IV, Cohomology of Groups, is motivated by the problem of group extensions, and crossed homomorphisms and factor sets lead naturally to the construction of the Bar Resolution. Chapter V, Tensor and Torsion Products, gives a new intrinsic definition of Tor (due to MacLane), and is applied to the Künneth Formula. Chapter VI on Types of Algebras defines algebras by diagrams, and discusses in detail graded algebras, differential graded algebras, coalgebras and Hopf algebras. Chapter VII, Dimension, defines homological dimension and includes among other things already noted, brief mention of some of the applications of homological dimension to local rings. Chapter VIII on Products includes a fairly complete discussion of the various types of homology and cohomology products and considers acyclic models and the Eilenberg-Zilber Theorem. In Chapter IX, Relative Homological Algebra, abelian categories are introduced, and resolutions relative to an allowable family of exact sequences are discussed. These results are then applied to Chapter X. Cohomology of Algebraic Systems, in which one finds a description of the Bar Resolution for algebras, and the cohomology of commutative DGA-algebras. Chapter XI, Spectral Sequences, contains a lucid exposition of spectral sequences. Chapter XII, Derived Functors, treats derived functors in great generality, discusses universality, connected pairs of additive functors, and the spectral Künneth formula.

Historical notes are found at the ends of some sections and chapters; there are helpful exercises at the end of each section, and the bibliography is fairly extensive.

It should be pointed out that the role of resolutions in this book is secondary. They are introduced after the definitions of Ext and of Tor as computational devices. In MacLane's approach they are not (and should not be) basic to the definitions.

There is a great temptation to ask that this book include many more applications in many more areas than it does. However, if we keep in mind the fact that the title of the book is *Homology*, and that the author assumes that we all know by now why we should study homology, we can appreciate the selectivity which the author has displayed. In the light of recent developments, Chapters IX and XII may well be a little out of date, but MacLane points out in his introduction that the subject is still in a state of flux, and it's anybody's guess as to what the final word may be (if any). In any event, this is a book which can be given to a student with the assurance that if he absorbs the material in it he will have learned a great deal. Moreover, the students who have been reading this book have found it to be extremely helpful and clear.

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Introduction to differentiable manifolds. By Louis Auslander and Robert E. MacKenzie. McGraw-Hill, New York, 1963. 9+129 pp. $9.95.

There seem to be two main routes toward an understanding of contemporary differential geometry and its allied disciplines. One way proceeds in historical order through the classical material on curves and surfaces in three-space, then on to global problems and the full development of manifold theory. The second approach starts in higher dimensions, leading directly from Euclidean spaces to the fundamentals of manifold theory. Although several good books have appeared recently following the first approach (for example, those by Willmore and Guggenheimer), there is a vital need for material presenting the intuitive background for the second. This book is a very welcome contribution to this goal, presenting in clear readable form of the basic concepts of the geometry of manifolds.

Chapters 1 and 2 introduce differentiable manifolds and their tangent spaces, proceeding from Euclidean spaces to submanifolds of Euclidean spaces and then on to abstract manifolds. To give further motivation for introducing the abstract objects the next chapter introduces the non-singular projective algebraic varieties as mani-