study any divisible semigroup, we need consider all congruences of $\bar{R}=\prod_\alpha R_\alpha$. For this purpose the following general result is used: A congruence of a commutative cancellative semigroup S is determined by a system of ideals of S and a system of subgroups of the quotient group of S.

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UNIVERSITY OF CALIFORNIA, DAVIS

ALMOST LOCALLY FLAT EMBEDDINGS OF S^{n-1} IN S^n

BY J. C. CANTRELL

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1. **Introduction.** In this paper we use the terminology introduced by Brown in [2]. We consider an $(n-1)$-sphere S embedded in S^n and try to determine if the components of S^n-S have closures that are n-cells (i.e. if S is flat). Brown has shown that if S is locally flat at each of its points, then S is bi-collared [2]. Hence, in this case, S is flat. The principal result of this paper is that if S is not flat in S^n, $n>3$, and E is the set of points at which S fails to be locally flat, then E contains more than one point. This is a fundamental point at which the embedding problems for $n>3$ differ from those for $n=3$. Throughout this paper we will assume that $n>3$.

2. **Outline of proof of principal result.** By combining Theorem 1 of [2] and Theorem 2 of [1] one can establish the following.

LEMMA 1. *Let S be an $(n-1)$-sphere in S^n and G a component of S^n-S. If S is locally collared in* Cl G, *then S is collared in* Cl G *and* Cl G *is an n-cell.*

For $0 \leqq t \leqq 1$ we let A_t be the solid ball in E^n which is centered at the origin and has radius t. Let B be the solid ball in E^n which is centered at -1 on the x_n-axis and has radius 2. With the aid of Theorem 1 of [2] we are able to establish the following lemma.

LEMMA 2. *Let S be an $(n-1)$-sphere in S^n, $p \in S$, and G and H the components of $S^n - S$. If S is locally flat at each point of $S-p$ and S has a local collar in* Cl H *at p, then there is a homeomorphism f carrying* $\mathrm{Cl}(B - A_{1/2})$ *into S^n such that* (1) $h(\mathrm{Bd}\, A_1) = S$, (2) $h[(0, 0, \cdots, 0, 1)] = p$, *and* (3) $h(\mathrm{Bd}\, A_{1/2}) \subset H$.

We keep the notation of Lemma 2 and let L be the closed arc in the x_n-axis from $(0, 0, \cdots, 0, 1/2)$ to $(0, 0, \cdots, 0, 1)$, and set $L' = f(L)$. There is a continuous mapping h of $\mathrm{Cl}(B - A_{1/2})$ onto itself such that h is the identity on Bd B, $h(\mathrm{Bd}\, A_{1/2}) = \mathrm{Bd}\, A_1$, and h carries $\mathrm{Cl}(B - A_{1/2}) - L$ homeomorphically onto $\mathrm{Cl}(B - A_1) - (0, 0, \cdots, 0, 1)$. Thus, if K is the component of $S^n - f(\mathrm{Bd}\, A_{1/2})$ which contains G, then there is a continuous mapping g of Cl K onto Cl G which carries (Cl K) $- L'$ homeomorphically onto (Cl G) $- p$. We keep in mind that Cl K is an n-cell and observe that the following statement is true. If there is a continuous mapping k of Cl K onto Cl K such that $k(L') = f[(0, 0, \cdots, 0, 1/2)]$ and k carries (Cl K) $- L'$ homeomorphically onto (Cl K) $- f[(0, 0, \cdots, 0, 1/2)]$, then Cl G is an n-cell (kg^{-1} is a homeomorphism of Cl G onto Cl K).

Thus in order to conclude that Cl G is an n-cell (and, hence that S is flat) it suffices to construct the mapping k above. If Cl K and L' are polyhedral there is no difficulty. So we assign to Cl K a combinatorial triangulation and proceed to move L' onto a polyhedral arc in Cl K. By results of Homma and Gluck [4] we may construct a homeomorphism r_1 of Cl K onto itself so that $r_1(L')$ is locally polyhedral at each point of $r_1(L' - p) = r_1(L') - r_1(p)$. Then Lemma 2 of [3] is used to obtain a homeomorphism r_2 of Cl K onto itself so that $r_2 r_1(L')$ is polyhedral. Then the desired mapping k can be constructed. These results give the following theorem.

THEOREM 1. *If S is as in Lemma 2, then S is flat.*

With an adaptation of Mazur's technique [5] we are able to remove the requirement of local collars in Cl H at each point of S and establish the following theorem.

THEOREM 2. *Let S be an $(n-1)$-sphere in S^n, $p \in S$, and G a component of $S^n - S$. If $S - p$ is locally collared in* Cl G, *then* Cl G *is an n-cell.*

COROLLARY. *If S is an $(n-1)$-sphere in S^n, $p\in S$, and S is locally flat at each point of $S-p$, then S is flat.*

3. **Conjectures.** If S is nonflat in S^n, $n>3$, and E is the set of points of S at which S fails to be locally flat, we have seen that E must contain more than one point. The natural question is: how many points must E contain? It is conjectured that there are no isolated points of E, and therefore E contains a Cantor set.

We say that the k-cell D in E^n is flat if there is a homeomorphism h of E^n onto itself such that $h(D)$ is a standard unit cell in the hyperplane $x_n=x_{n-1}=\cdots=x_{k+1}=0$. The author has reduced the above conjecture to the following.

CONJECTURE. If $D=D_1\cup D_2$, where D_1 and D_2 are flat $(n-1)$-cells in E^n, $n>3$, and $D_1\cap D_2=\text{Bd } D_1\cap\text{Bd } D_2$ is a flat $(n-2)$-cell, then D is flat.

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