ure on the line. Chapter VII is devoted to the work of the Polish school on compact and quasi-compact measures, and Chapter VIII to conditional probabilities.

DOROTHY MAHARAM

Stability by Liapunov's direct method with applications. By Joseph La Salle and Solomon Lefschetz. Academic Press, New York, 1961. vii+133 pp.

The stability criteria of Lyapunov's Second or Direct Method have been largely unknown to engineers in this country until very recently. The fact that they begin to be used now is in no small measure due to the authors' persistent efforts to acquaint engineers and mathematicians alike with this area of stability theory, which was developed almost entirely in the Soviet Union. The present monograph, the first on this subject written in English, is an outgrowth of these efforts. It is frankly aimed at the engineer with modest mathematical background. Thus, the principal results of Lyapunov's stability theory are presented with a minimum of technical detail; proofs of theorems when given are formulated in geometric rather than analytic language; examples selected with a view towards applications are completely worked out in the text. The pace is leisurely throughout, the exposition uncluttered and easily readable.

Chapter 1, entitled "Geometric concepts: Vectors and matrices," contains introductory material on vectors, matrices, quadratic forms and Euclidean geometry needed in the sequel. Chapter 2 on "Differential equations" forms the core of the book. Here the authors introduce the concepts of stability for a solution of an autonomous vector equation, define Lyapunov functions for such equations and prove the classical theorems of Lyapunov and Cetaev. The corresponding theorems for nonautonomous equations are mentioned only briefly. The construction of Lyapunov functions is illustrated in a number of examples of equations of second order and systems of first order, including the so-called critical case. A thorough discussion of the regions of stability, here called the extent of stability, follows. It includes proofs of the important generalizations of Lyapunov's theorems on asymptotic stability which do not require negative-definiteness of the time derivative. Chapter 3, entitled "Application of Liapunov's theory to controls," takes up a detailed study of linear control systems with continuous servo characteristic. Explicit stability criteria are derived from a suitable Lyapunov function whose construction is carried out in several cases. In Chapter 4, concerned with "Extensions of Liapunov's method," the authors discuss the use of differential inequalities in stability considerations, theorems on the ultimate boundedness of solutions and a concept they introduce called "practical" stability which fits more closely the requirements arising in technical applications than the usual notion of stability. This point is illustrated on van der Pol's equation with a small perturbation term.

Mathematicians may object to a few statements such as the definition of a closed set as the outside of some open set (p. 19). In the reviewer's opinion these are unfortunate attempts to please the potential reader by keeping the mathematical details to a minimum. Such statements could have been avoided without detracting from the appeal this book will unquestionably have with the audience for which it is primarily intended.

H. A. ANTOSIEWICZ

Die innere Geometrie der metrischen Räume. By Willi Rinow. Springer, Berlin, 1961. 16+520 pp. DM 83.

The intrinsic geometry of metric spaces studies their invariants under isometries. Its history dates from Gauss' Theorema Egregium, and it developed in the nineteenth century under Riemann, Helmholtz, Lie, and Klein. In this century intrinsic *differential* geometry has been pursued with great vigor, quite apart from the larger context. In fact, despite great successes in general metric geometry achieved in the past thirty years, mathematical fashion has all but ignored it as a bona fide field for research.

Recently, however, the climate has begun to change. Interest in differential geometry in the large has started many a mathematician studying intrinsic metric geometry. But such a study is made difficult by the scarcity of books on the subject. Until the present volume appeared, there were only three in the field: A. D. Aleksandrov's, *Intrinsic geometry of convex surfaces* (Moscow, 1948), L. M. Blumenthal's, *Theory and applications of distance geometry* (Oxford, 1953), and H. Busemann's, *Geometry of geodesics* (New York, 1955).[1] Since these treat restricted aspects of the whole subject, some survey which could help describe and define the field was clearly called for.

Rinow's book apparently was planned as just such a survey. However, as he states in the introduction, the lack of uniformity in assumptions and definitions among writers in the field soon made it

[1] A predecessor of Busemann's book, his monograph, *Metric methods in Finsler spaces and in the foundations of geometry* (Princeton, 1942), should also be mentioned.