BOOK REVIEWS

Lie algebras. By Nathan Jacobson. Interscience Tracts on Pure and Applied Mathematics, no. 10. Interscience Publishers, New York, 1962. 9+331 pp. $10.50.

Although Lie algebras are about 80 years old one might say that they are now barely past their adolescence. They appeared originally as "infinitesimal groups," i.e., as objects derived from Lie groups for the purpose of linearizing some of the basic group theoretical problems. However, it soon became evident that the resulting linear problems concerning the structure and the representations of Lie algebras were difficult and of a quite novel type, so that the algebraic technique existing at the turn of the century was inadequate to the tasks thus imposed on it.

It was this very difficulty which led to some of the most beautiful and most fruitful developments in linear algebra, the decisive steps being taken (in this order) by W. Killing, E. Cartan and H. Weyl (1888–1926). Since the methods used involved analysis and topology, it took several decades more—right up to the present—to put Lie algebra theory on a firm purely algebraic foundation and to extend the results to base fields other than that of the real or complex numbers. It is in this phase of Lie algebra theory that the author of the present book has played an important role. What he presents here is the first essentially complete and selfcontained exposition of the main results on the structure and the representations of Lie algebras which, moreover, embodies many original and substantial simplifications of the theory.

There are ten chapters, each of which begins with a brief description of its content and ends with a series of exercises designed so as to either supplement the theory in certain points or to develop some of the requisite algebraic technique. Knowledge of elementary linear algebra is presupposed and is a sufficient prerequisite for the study of all but the last chapter, where Galois theory and some of the Wedderburn structure theory of associative algebras is needed in addition.

It is a common feature of all existing structure and representation theory of algebras and groups that considerable heights have been attained for semisimple algebras and groups, while—in the general case—not much more has been achieved than the splitting off of the semisimple part from the radical and the superficial control of the solvable case. In this book, the main peaks of the semisimple theory are reached in Chapters IV and VII. Here, it is assumed that the base field is of characteristic 0 and that the semisimple Lie algebra is "split" in the sense that it has a decomposition into root spaces with respect to some Cartan subalgebra, which is a relaxation of the usual assumption that the base field be algebraically closed. Chapter IV gives the classification (in terms of the Dynkin diagrams of systems of simple roots) of the split semisimple Lie algebras, and Chapter VII gives the classification of the irreducible finite dimensional representations of these Lie algebras in terms of the weights with respect to a Cartan subalgebra. The representation theory of these Lie algebras is continued in Chapter VIII, which gives Weyl's character formula and its consequences; namely Kostant's formula for the multiplicity of any weight in a given irreducible representation and Steinberg's formula for the multiplicity with which any irreducible representation occurs in a tensor product of two given irreducible representations. (That these are consequences of the character formula was discovered only recently by Cartier and Steinberg, respectively.) The structure theory is continued in Chapters IX and X. Chapter IX begins with the Cartan-Chevalley conjugacy theorem for Cartan subalgebras of arbitrary Lie algebras over an algebraically closed field of characteristic 0 and then gives the determination of the automorphism groups of the simple Lie algebras over such fields. Chapter X discusses the classification of the simple Lie algebras over arbitrary fields of characteristic 0, using the centroid as the base field and analyzing what happens when the Lie algebra is tensored with a Galois extension of the centroid.

The other chapters, I, II, III, V, VI, deal with the general theory and furnish the tools for what has been mentioned above. Roughly, this is the same material which is covered (in part only by exercises) in N. Bourbaki's *Groupes et algèbres de Lie*, Chapter I.

Two more comments on the content are called for. The first is that the important theory of algebraic Lie algebras is not touched in this book, despite its intimate connections with the general theory. The excuse for this omission is that a thorough exposition of it is available from the horse's mouth (C. Chevalley, *Théorie des groupes de Lie*, vol. III) and that its inclusion would have swelled this book to a formidable size. The second comment is aimed in the opposite direction: the author has, with obvious (and fully justifiable) reluctance, given in to a feeling of responsibility toward the cohomology theory of Lie algebras. This has resulted in two fleeting excursions into that theory, a short one in Chapter III and a considerably longer one in Chapter V. However, in the absence of a background of general concepts of homological algebra, these are more misleading than helpful, and it would have been better to omit them altogether. Indeed, it is by now quite visible that cohomology theory is not very useful as a tool for either the structure theory of algebras or groups or for the representation theory. Cohomology theory is more appropriately regarded as a superstructure built upon representation theory and feeding on it.

Occasional lapses of style call for a word of warning to the novice, because they are features not suitable for emulation. Thus, on p. 209, there is introduced a notational convention based on the barbarous principle of confusing a function with one of its values. The resulting blurs on otherwise beautifully clear proofs recur on a number of subsequent pages. More startling is the discussion of polynomial functions on a vector space given on p. 266. This is evidently addressed to a beginner rather than to the reader who has followed so far. The underlying principle here is the preference of a complicated non-invariant definition over a simple invariant one. The introductory discussion on p. 295 is of a similar nature; here it seems that an effort was made to hide the nature of a tensor product.

For a peaceful conclusion, let it be said that, in gratitude for what this text offers, one is more than ready to forgive the author for his rare nostalgic returns to a primitive mathematical language. In essence, this book is a superbly well organized, clear and elegant exposition of Lie algebra theory, shaped by the hands of a master. It remedies what has been an exasperating deficiency in the mathematical literature for many years.

G. Hochschild

Lectures on the theory of functions of a complex variable, Vol. I. By Giovanni Sansone and Johan Gerretsen. P. Noordhoff Ltd., Groningen, 1960. 12+481 pp. Paper, $12.00. Cloth, $13.00.

In 1947 Professor Sansone published his *Lezioni nulla teoria delle funzioni di una variabile complesa* in two volumes of 359 and 564 pages respectively (reviewed in Bull. Amer. Math. Soc. **54** (1948)). The present volume is a completely new text, based on the Italian edition.

The chapter headings are:

1. Holomorphic functions. Power series as holomorphic functions. Elementary functions. (40 pages)

2. Cauchy's integral theorem and its corollaries. Expansion in Taylor series. (81 pages)

3. Regular and singular points. Residues. Zeros. (63 pages)

4. Weierstrass's factorization of integral functions. Cauchy's ex-