such a continuum in a paper, in Russian, entitled *On the set of boundary values of meromorphic functions*, which appeared in 1952 in the Doklady of the Soviet Academy.

Another section is devoted to the notion of Baire category, as applied to the theory of cluster sets, which in recent years was developed by Bagemihl and the reviewer and by Collingwood. Among other results appearing here are the well-known theorems of Plessner and of Lusin and Privalov, as well as the "two-chord" theorem of Meier. It is not clear to the reviewer why the latter is referred to as a sharpening of Plessner's theorem.

The final section of Chapter III studies the boundary behavior of functions of bounded type, investigated recently by Lehto, and of normal functions. In connection with functions of bounded type, reference should have been made to Gehring's interesting paper which appeared in the Quarterly Journal of Mathematics in 1958. The class of normal functions was first introduced by the author in an undeservedly little known paper in 1939, in which he also derived some important properties of this interesting class of functions. Subsequently, in 1957 Lehto and Virtanen independently rediscovered these functions and made significant contributions to their theory.

Two omissions in Chapter III should be noted. The Uniqueness Theorem 7 in §3 is very closely related to Corollary 3 of the first of Bagemihl's papers listed in the bibliography, but no mention is made of this. Also, no mention is made of the fact that Theorem 15 in §3 was proved independently by Bagemihl and the reviewer in the paper listed as [5] in the bibliography.

The last chapter goes into extensions of the theory of cluster sets to single-valued analytic functions on open Riemann surfaces, and a short appendix touches on extensions to pseudo-analytic functions. The book ends with an excellent bibliography.

The author is to be congratulated for his concise, but very readable, survey of a highly technical and extensive subject. It can be warmly recommended to any one who wishes to become acquainted with this branch of function theory.

W. SEIDEL

Funktionalgleichungen der Theorie der geometrischen Objekte. J. Aczél and S. Gołab. Panstowe Wydawnictwo Naukowe, Warsaw, 1960. 172 pp. DM 36.00.

In the 1930's a rather interesting correspondence took place between the Dutch differential geometers J. A. Schouten and J. Haantjes on one side, and their Polish colleague A. Wundheiler on the other. The question at stake was what the most general (local) structures would be on a differentiable manifold, and how a theory of such things could be built up. A motivating factor in their discussion was the fact that—by the example of affine connections—tensors were not sufficiently general to cover all possibilities. The result, published in 1936-1937, was the definition of objects, macro-geometric objects and micro-geometric objects. All three have in common with tensors that finite ordered sets of numbers (or functions)—called components—are assigned to each admissible coordinate system. Objects were defined to be just that, while for the geometric objects the components with respect to two coordinate systems must be obtainable from each other if the relation between the coordinate systems is known. Roughly speaking, the macro-geometric objects remind us of sheaves, while the micro-geometric objects are reminiscent of fibre bundles. Most later literature, including the book under review, deals with micro-geometric objects only, and calls them geometric objects.

While attempts to formulate the theory of geometric objects in fibre terminology were made by Haantjes-Laman (1953), Kuiper-Yano (1955), and one with functorial overtones was made by the reviewer (1960), practically all of the 120 or so works on the subject (three-fourths of which by 8 authors) use the original terminology of Schouten-Haantjes and Wundheiler. To modern mathematicians this may seem cumbersome, but changing the terminology would not solve any hard problems, of course.

Really very little is known about geometric objects. The broad generalities are contained in the reviewer's thesis (1952), while the very special facts and results are the main concern of the book under review. The outstanding problem, according to the authors, is that of classification, to which essentially the whole book is devoted. A brief summary of generalities and definitions is given in the first chapter, including that of geometric object of type (m, n, r): m is the number of components, n the dimension of the underlying manifold and r the highest order of derivatives of coordinate transformations needed to express the components with respect to one system in terms of the components with respect to another. Chapter II discusses the so-called nondifferential ($r=0$) and purely differential g.o., and classifies certain g.o. of types $(1, 1, r) r \geqq 4$, $(1, 1, 2)$, $(1, 1, 3)$, $(1, 1, 1)$, $(1, n, 1)$, $(m, 1, r) r \leqq 3$, and few other situations. Chapter III, on algebra of g.o., deals with g.o. which are functions of others (in a manner independent of coordinates), in particular a generalized addition procedure and a discussion of type $(1, n, r)$. In Chapter IV fields of g.o. are considered, and covariant differentiation is introduced, in particular for type (1, 1, 1), (1, 1, 2), (m, 1, 1) and (m, 1, 2). The last chapter discusses further questions such as tensor comitants of tensors, Lie derivatives and some unsolved problems.

A number of results in the book are products of considerable ingenuity. Yet, anything even remotely resembling a complete classification is much too optimistic an aim. Geometric objects are just too easy to manufacture: take any eager student who knows the summation convention and other rites of tensor arithmetic and let him stir some tensor fields and partial differential operators; affine connections to be added according to taste. The result is invariably an object and chances are that by adjoining some intermediate steps of the random computation (for example, everything he has written down) one obtains a geometric object. On these grounds the reviewer believes that no classification attempts make much sense until, by suitable restrictions, the trivial has been separated from the significant. For example, it seems very unlikely that limiting the dimension n of the underlying space to values such as 1 or 2 is really much help in this respect.

It is the reviewer's personal opinion that geometric objects will never be of as central a significance as their inventors had hoped. On one hand, they serve as a suitable context in which to formulate general procedures (such as Lie differentiation); on the other hand, certain well-selected sub-classes will be worth classifying. One hint in this direction is Wagner's result (1950) on simple geometric objects; another is opened up by the discovery of new tensor differential comitants of tensor fields in recent years (the book mentions both topics only casually). The first of these accomplishments has singled out affine connections as very special geometric objects; the second has given us machinery to deal with (almost)-complex structures and their deformations. Thus, one may consider these areas as significant, and try to judiciously expand from here to a broader class of geometric objects which have some similar promise. As in other parts of mathematics, one will also here have to sail between the Scylla of too stern a utilitarianism and the Charybdis in which the useful is flooded by the trivial.

Besides noticing the "classical" language of differential geometry of the 1930's (which is basically as good as any other) the reader will also be struck by some conceptual inaccuracies of the same era. Thus, n-dimensional manifolds are defined as sets topologically equivalent to subsets (no openness required) of R^n (p. 1), yet partial derivatives of coordinate n-tuples are cheerfully computed (p. 2). It is claimed that for equivalent geometric objects (i.e. the components of each are functions of those of the other in a manner independent of coordinates) the numbers of components are the same (p. 16), without any continuity hypotheses. Also, the discussion of the domains of various functions (especially those giving the transformation of components under a change of coordinates) is not satisfactory. The reviewer may hardly seem entitled to make these criticisms of terminological and conceptual casualness in view of a similar carelessness in his own 1952 work; yet he cannot help notice with regret that the present authors have made no apparent attempt to meet the standards of clarity and precision achieved in the last decade.

ALBERT NIJENHUIS

Combinatorial topology. Vol. 3. By P. S. Aleksandrov. Translated by Horace Komm. Graylock Press, Albany, N. Y., 1960. 8+148 pp. $6.50.

This Vol. 3 is the English translation of the last two parts, Parts IV and V, of Aleksandrov's *Kombinatornaya topologiya* (OGIZ, Moscow-Leningrad, 1947). The English translation of Parts I–III has been previously published in Vols. 1 and 2 (See the Review in this Bulletin, **62** (1956), 629–630; **64** (1958), 300–301). The present Vol. 3 is devoted to two topics of homology theory: the Alexander-Pontrjagin duality theorem and an introductory theory of mappings of polyhedra.

Part IV consists of Chapters XIII–XV. Homological manifolds are introduced in Chap. XIII, together with preliminaries needed for the proof of the Alexander-Pontrjagin duality theorem. At the same time, the Poincaré-Veblen duality theorem is proved. Chap. XIV begins with Čech cohomology groups of compact Hausdorff spaces, and then proceeds to the proof of the Alexander-Pontrjagin duality theorem. In Chap. XV, the original Alexander duality theorem is given a separate proof, which is independent of Chap. XIV and is preceded by a discussion of linking in Euclidean spaces.

Part V is divided into two chapters. Chap. XVI presents several classical elementary theorems on mappings. One finds here Poincaré-Bohl's theorem, Brouwer fixed point theorem, theorems on continuous vector fields, and, most important of all, Hopf's classification theorem on mappings of an n-sphere into another. The final Chap. XVII, devoted to the Lefschetz-Hopf fixed point theorem, is a revision of Chap. 14 of Alexandroff-Hopf's *Topologie* I (Springer, Berlin, 1935).

The material of this volume is well chosen. All the theorems are important, of classical nature and have great esthetic appeal. The