The purpose of the book under review is to arrive at the theory of the spinor genus by elementary methods, i.e. using matrix calculus and assuming some knowledge of elementary number theory. Whatever is needed from the arithmetic theory of quadratic forms is defined and proved. The development takes place in three main parts: local and global field equivalence, integral p-adic equivalence and the genus, and finally the spinor genus.

Chapters 1–3 give the Hasse-Minkowski theory of fractional equivalence. The fundamental result that a global form represents 0 if and only if it does so locally everywhere is proved by reduction theory and Dirichlet's theorem on primes in an arithmetic progression.

Chapters 4–5 discuss local integral theory and the genus. Included is an important theorem on the integers represented by an indefinite form in at least four variables.

Chapters 6–8 do the spinor genus. The difficult part here is to establish the relations between cls f, spn f, and gen f. Much time is also spent on rounding out the picture with additional results, some of them new.

The book suffers from an exasperating conceptual deficiency. Too often a formula or device is used to circumvent the introduction of an idea. Surely there is no longer any need to shy away from groups and vector spaces. And in a field that is so intimately concerned with questions of linearity, is it right to do so? Isn't it really better, and indeed simpler, to use p-adic numbers instead of families of congruences? We can sympathize with the author's efforts to keep out superfluous structure, but the criterion used in doing so should be conceptual, not just logical, necessity.

However, these are small matters. The important thing is that the author has contributed an ordered account of significant results in a field with a long history and a totally inadequate literature. The book will be read by people interested in quadratic forms, and it should provide an accessible reference for those who are interested in the applications.

O. T. O'MEARA

Fondements de la topologie générale. By Ákos Császár. Akadémiai Kiadó, Budapest, 1960. 231 pp. $6.00.

The author's goal is to treat uniform, proximity, and topological spaces from a common viewpoint. He accomplishes this by developing a very general theory of "syntopogenic structures" in which uniformities, proximities, and topologies emerge as particular cases. The idea is simple and interesting. A syntopogenic structure on a set E is a family of (partial) ordering relations on $P(E)$ satisfying certain natural conditions. The family defining a topology, for example, consists of a single relation $<$, with $A<B$ meaning that A is contained in the interior of B.

The subject is presented in exhaustive detail, but an excellent summary is included in the introduction. Formal prerequisites are minimal. Starting with a review of elementary facts about relations in general, the book continues through sixteen sections of painstaking discussion of "semi-topogenic," "topogenic," "perfect," "biperfect," "simple," and "symmetric" orders or structures, and of generalizations to them of the definitions and main theorems about continuity, product spaces, separation axioms, convergence of filter bases, completion, and compactification. For example, arbitrary products of complete [compact] syntopogenic structures are complete [compact]. There is even the theorem, due to J. Czipszer, that every syntopogenic structure is induced by a family of "upper semi-continuous" real-valued functions, i.e., functions continuous with respect to a structure on R defined in terms of $<_\epsilon$ $(\epsilon>0)$, where

$$A <_\epsilon B \text{ if } \sup A + \epsilon \leqq \inf (R - B). \tag{a}$$

One who wishes to investigate the subject further will find the basic work all done, formally recorded, and easy to locate. The book contains a helpful index of terminology, axioms, and special notation, the last containing nearly 100 entries. The numbering scheme for theorems and displays is simple, and the numbers are included with the section titles and numbers in the page headings. Most of the facts cited in the proofs are accompanied by back references. These aids also allow a reader to pass over much of the routine spadework, knowing that he will be able to find things quickly if he should ever need them.

The rest of this review presents details leading to the familiar structures. Let E be a set. A *topogenic order*—for brevity, an *order*—on E is a relation $<$ on $P(E)$ satisfying:

$$0 < 0, \qquad E < E, \tag{1}$$

$$A < B \quad \text{implies} \quad A \subset B, \tag{2}$$

$$A \subset A' < B' \subset B \quad \text{implies} \quad A < B \tag{3}$$

(so far, a "semi-topogenic" order), and

$$A < B \text{ and } A' < B' \text{ implies } A \cup A' < B \cup B' \text{ and } \\ A \cap A' < B \cap B'. \tag{4}$$

By (2), $A<B$ and $B<A$ implies $A=B$; by (2) and (3), $<$ is transitive.

To see how such an order may be generated, consider a family of subsets of E that is closed under finite union and finite intersection (i.e., a sublattice of $P(E)$ containing 0 and E)—for brevity, a *lattice*. Each lattice $\mathcal{G}$ generates an order $< = O(\mathcal{G})$ [reviewer's notation] defined by

(b) $\qquad A < B$ if there exists $G \in \mathcal{G}$ such that $A \subset G \subset B$;

it then follows that

(c) $$\mathcal{G} = \{G: G < G\}.$$

Conversely, each order $<$ determines a lattice $\mathcal{G} = L(<)$ [reviewer's notation] defined by (c). Furthermore,

(d) $$L(O(\mathcal{G})) = \mathcal{G}$$

and

(e) $$O(L(<)) \subset <.$$

The inclusion in (e) may be proper. Thus, every lattice is induced by an order, but there exist orders that are not generated by any lattice —for example, the order $<_\epsilon$ defined in (a).

An order $<$ is said to be *perfect* if $A_i < B_i$ implies $\bigcup_i A_i < \bigcup_i B_i$ (over arbitrary index sets), *biperfect* if the preceding holds as well for intersections, *symmetric* if $A < B$ implies $E-B < E-A$. (Note that this is not the same as symmetry of $<$ as a relation.) Trivially, perfect and symmetric implies biperfect. It is clear that these types of orders are quite special. Indeed, the biperfect orders $<$ on E are in one-one correspondence with the reflexive relations U on E, according to:

(f) $$x \, U \, y \text{ if and only if } \{x\} \nless E - \{y\}$$

and

(g) $\qquad A < B$ if and only if $x \in A$ and $x \, U \, y$ implies $y \in B$.

(Thus, the biperfect ordering relations on the subsets of E are expressible in terms of the elements of E.)

A *syntopogenic structure*—for brevity, a *structure*—on E is a family $\mathcal{S}$ of orders directed by $\subset$ and such that each member is contained in the square of some member, i.e.:

(5) $$\begin{gathered} \text{for } <' \in \mathcal{S} \text{ and } <'' \in \mathcal{S}, \text{ there exists } < \in \mathcal{S} \text{ such that} \\ A <' B \quad \text{and} \quad A <'' B \quad \text{implies} \quad A < B, \end{gathered}$$

and

$$\text{for } < \in \mathcal{S}, \text{ there exists } <' \in \mathcal{S} \text{ such that } A < B \text{ implies} \quad A <' C <' B \text{ for some } C. \tag{6}$$

A structure is called perfect, biperfect, or symmetric in case all its members are perfect, biperfect, or symmetric, respectively. A structure with just one member is *simple*. (Here, (6) reduces to: $<^2 = <$.) The family $\{\subset\}$ is an obvious example of a simple, perfect, symmetric structure. Conversely, if $\{<\}$ is perfect and symmetric, then the associated relation U of (f) proves to be an equivalence (transitivity following from (6)), and $</U$ is set inclusion on E/U.

The familiar structures arise from taking the three main conditions two at a time:

simple and perfect $\leftrightarrow$ topology,

simple and symmetric $\leftrightarrow$ proximity,

perfect and symmetric $\leftrightarrow$ uniformity.

Specifically, if $\{<\}$ is perfect, then the family $\mathcal{G}$ of (c) is evidently a topology on E; moreover, it turns out (with the help of (6)) that equality holds in (e). Conversely, if $\mathcal{G}$ is a topology, then the order generated by $\mathcal{G}$ is perfect. Thus, there is a one-one correspondence between topologies and simple perfect structures on E. Note that $A < B$ means that B is a neighborhood of A.

Next, if $\{<\}$ is symmetric, define $A\delta B$ ("A is near B") to mean $A \not< E-B$. Then $A\delta B$ implies $B\delta A$; $(A\cup B)\delta C$ if and only if $A\delta C$ or $B\delta C$; $\{x\}\delta\{x\}$; $A\bar{\delta}0$; if $A\bar{\delta}B$, there exist U, V such that $A\bar{\delta}(E-U)$, $B\bar{\delta}(E-V)$, and $U\cap V=0$. These are the axioms for a general proximity. (To ensure that $x\neq y$ implies $\{x\}\bar{\delta}\{y\}$, as required in Efromovic's definition of proximity, one introduces a "T_0" separation axiom: if $x\neq y$, then $\{x\} < E-\{y\}$ or $y < E-\{x\}$.) Conversely, given a general proximity δ, let $A < B$ mean $A\bar{\delta}(E-B)$; then $\{<\}$ is a symmetric structure. The simple symmetric structures are in obvious one-one correspondence with the general proximities on E.

Finally, consider a perfect symmetric structure $\mathcal{S}$. For each $<\in\mathcal{S}$, consider the relation $U=U_<$ defined in (f); let $\mathcal{U}$ denote the family of all such U. Then each member of $\mathcal{U}$ is reflexive and symmetric, $\mathcal{U}$ is directed by $\supset$, and each member of $\mathcal{U}$ contains the square of some member. Therefore $\mathcal{U}$ is a base of symmetric entourages for a uniformity. Conversely, let $\mathcal{U}$ be a symmetric base for a uniformity. For each $U\in\mathcal{U}$, define $<$ by (g); then the family of all such $<$ is a perfect symmetric structure. The correspondence between symmetric bases and perfect symmetric structures is one-one. Note that $A < B$ means that B contains the neighborhood of A of order $U_<$.

The passage from uniformity to proximity to topology goes this way. If $\mathcal{S}$ is perfect and symmetric, then $\{<\} = \{\cup \mathcal{S}\}$ is (simple and) symmetric; and if $A <' B$ means that $\{x\} < B$ for all $x \in A$, then $\{<'\}$ is (simple and) perfect.

The familiar discrete structures are obtained from the family $\{\subset\}$. The usual uniformity on R is obtained from $\{<^\epsilon : \epsilon > 0\}$ [reviewer's notation], where $A <^\epsilon B$ means dist $(A, R-B) \geqq \epsilon$. (The associated relations U^ϵ of (f) then satisfy: $xU^\epsilon y$ if and only if $|x-y| < \epsilon$.)

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RESEARCH PROBLEM

28. Frank Harary. *Matrix theory.*

Prove or disprove the following conjecture suggested by J. Selfridge (oral communication). For any graph G with 9 points, G or its complementary graph $\bar{G}$ is nonplanar. Experimental evidence appears to support this conjecture, which in turn would imply the validity of the conclusion for any graph with at least 9 points. A simple argument using Euler's polyhedron formula serves to prove that if G is a graph with p points and q lines for which $q > 3p-6$, then G is nonplanar. This proves the conclusion of the conjecture for all graphs with at least 11 points. For graphs G with 9 or 10 points, it is still open. (Received August 15, 1961.)