BOOK REVIEWS

Fourier transforms. By R. R. Goldberg. Cambridge University Press, 1961. viii+76 pp. \$3.75.

This short book contains an elementary exposition of the basic theorems of Fourier analysis on the real line. A rudimentary knowledge of Lebesgue integration is all that the reader is required to have.

The following topics are treated: Inversion and uniqueness (via $(C, 1)$-summability) of the L^1-Fourier transform, the action of analytic functions on Fourier transforms, Wiener's theorem on the span of the translates of a function f in L^1 (including some extensions in which the transform of f is allowed to have zeros), the Plancherel theorem, and finally Bochner's theorem on the representation of positive-definite functions as Fourier-Stieltjes transforms.

The author deliberately avoids any appeal to functional analysis in his proofs, which are quite explicit and constructive. He does, however, point out how the theory can be carried over to arbitrary locally compact abelian groups and he describes the algebraic setting of Wiener's theorem in terms of ideal theory. The maximal ideals of L^1 are identified. There are also references to some very recent papers.

The book may thus be described as a concrete introduction to abstract harmonic analysis.

WALTER RUDIN

An introduction to homological algebra. By D. G. Northcott, Cambridge University Press, 1960. xi+282 pp. \$8.00.

This book is a leisurely and detailed introduction to homological algebra. Very little background is assumed and the account is essentially self-contained.

The author begins by explaining the basic ideas concerning modules over a ring, functors and categories, as well as treating the tensor product and Hom in some detail. Next the homology functor is introduced, the connecting homomorphism is defined and its standard basic properties are given. After introducing projective and injective modules, resolutions of modules are discussed and then the theory of derived functors is developed. Next, the general theory of Ext and Tor is treated, the various homological dimensions are defined and their basic properties stated. This whole development is carried out only for the category of modules over rings.

The material sketched occupies about 140 pages. It seems to this reviewer that the author has fallen into the trap of believing that the more detail given, the more intelligible a subject becomes. For example, it takes four theorems with separate proofs for the author to say that $\mathrm{Tor}^{\Lambda}(A, B)$ may be computed from a projective resolution of A or from a projective resolution of B, or from using resolutions of both. The analogous four theorems are also given for Ext. The ideas of the proofs are essentially those to be found in Cartan-Eilenberg's *Homological algebra*. While the present treatment is much more detailed than that to be found there, it does not seem to this reviewer that it renders the subject more accessible. Indeed, because of the length of the text, it will be difficult for the novice to pick out those points that are really basic.

The next part of the book deals in the main with material not to be found in Cartan-Eilenberg: M. Auslander's result that l.gl.dim. is the supremum of the projective dimensions of cyclic left modules is given. Homological dimensions in general noetherian rings are discussed and the connection for these between a ring and its various rings of quotients are carefully worked out. The high point of this part of the book is undoubtedly the homological theory of local rings culminating in the Serre, Auslander-Buchsbaum theorem that a local ring is regular if and only if its global dimension is finite. The author has made this theory fairly self-contained by carefully stating all the ideal theoretic background necessary and supplying references to missing proofs.

The exposition of all these topics follows very closely that of the original research papers. No mention is made of the simpler proof of M. Auslander's theorem due to Eilenberg and to be found in Matlis' paper, *Applications of duality*, Proc. Amer. Math. Soc. vol. 10 (1960). Nor is Kaplansky's shorter proof of the Serre, Auslander-Buchsbaum theorem (University of Chicago Notes, 1959) referred to.

The final chapter of the book provides a brief introduction to the homology of groups. The standard resolution, interpretation of the first and second groups, the case of free abelian and nonabelian groups and monoids, and the complete derived sequence for finite groups are given very much as in Cartan-Eilenberg.

Thus, this volume treats a rather circumscribed area of homological algebra, the exposition being, however, not very different from that already available. Moreover, the usefulness of this book would have been increased by including a discussion of several points well within its scope. For example, although the text hints strongly at the fact that in general l.gl.dim. $\neq$ r.gl.dim. this point is never made explicit and Kaplansky's example of a ring R with r.gl.dim. $R=1$, l.gl.dim. $R=2$ (Nagoya Math. J. vol. 13 (1958)) is not cited. More serious, in this reviewer's opinion, is the lack of a complete bibliography; the inexperienced reader may remain unaware of much material in the literature, especially the kind that should certainly be given as suitable for further study. Thus, for example, no mention is made of Hochschild's relative theory, nor is most of the theory dealing with abstract categories and leading to a cohomology theory of sheaves mentioned. Since neither Godement's book nor Grothendieck's Tohoku journal papers are referred to, there is no indication of the many applications of homological algebra to topology and algebraic geometry.

Despite these drawbacks, it should be noted that in the topics treated, the author has given a very careful treatment of a relatively new subject. His work will certainly serve to disseminate these new ideas to a wide public.

ALEX ROSENBERG

Lectures on modern geometry. By B. Segre. With an appendix by L. Lombardo-Radice. Consiglio Naz. di Richerche, Monografie Matematiche. Roma, Ed. Cremonese, 1961. 15+479 pp.

The greater part of this book is a particularly lucid introduction in projective geometry, mainly over commutative fields, which goes as far as Plücker coordinates, invariants of projectivities, and the Schläfli-Berzolari theorem. Up to this point it is a new elaboration of the author's earlier *Lezioni di geometria moderna.* The last two chapters, however, contain new material, mainly due to the author himself, and published in several periodicals. The appendix, written by Lombardo-Radice gives an exposition of newer results on non-Desarguesian finite planes, of Moufang, Hall, Zorn and Levi, Gleason, Wagner and many others. Segre's research on finite planes, as put forth in the present work, is mainly concerned with the notion of k-arc, which is a set of k points no three of which are collinear. For a characteristic $\neq 2$, the $(q+1)$-arcs, (q=cardinality of the underlying field) are the irreducible conics, and every q-arc is contained in a uniquely determined conic (for $q \geqq 5$). There are, however, for characteristic $\neq 2$ maximal k-arcs which are not conics. They are extensively studied also for characteristic 2. Kustaanheimo's betweenness relation, generalizations of Menelaos' and Ceva's theorems, normal rational curves are other subjects. The only chapter dealing with non-pascalian geometries is devoted to reguli and their sections. Its main feature is an attempt on proving Wedderburn's theorem by geometrical means. Though the attempt was not successful (as stated