BOOK REVIEWS

Lectures on the theory of integral equations. By I. G. Petrovskii. Trans. from the second (1951) Russian Edition by H. Kamel and H. Komm. Rochester, Graylock, 1957. vi+94 pp. $2.95.

This monograph has been translated into very readable English by the translators and gives an excellent expository introduction to the theory of integral equations of the second kind:

$$\phi(P) = \lambda \int_G K(P, Q)\phi(Q)dQ + f(P) \tag{1}$$

where P, Q belong to G, a d-dimensional domain bounded by a finite number of pieces of $(d-1)$-dimensional surfaces and $f(P)$ is piecewise continuous. The parameter λ may or may not appear explicitly.

The elastic string is given as a motivating example. This is followed by approximating (1) by means of replacing the integral by finite Riemann sums. Solutions and conditions insuring solutions of this linear algebraic system are noted and, instead of passing to the limit, these algebraic results are used to suggest the Fredholm alternative theorems for the integral equation (1).

These theorems are then proved in turn for: (i) degenerate kernels $K(P, Q) = \sum_{i=1}^{m} a_i(P)b_i(Q)$, which reduce immediately to linear algebraic systems; (ii) sufficiently small uniformly continuous kernels, where (1) is solved by successive approximations obtaining a Neumann series (in powers of λ) involving convolutions of such kernels; (iii) almost degenerate kernels, which are sums of those in the preceding two cases; (iv) uniformly continuous kernels, which by the Weierstrass Theorem can be uniformly approximated with arbitrary accuracy by a degenerate kernel, i.e., a polynomial; and, finally, (v) kernels of the type $K(P, Q)/PQ^{\alpha}$, where $K(P, Q)$ is uniformly continuous, PQ is the distance between P and Q and $0 \leqq \alpha < d$.

Volterra equations are noted to be a special case of (v) and are dismissed with a brief discussion.

The remaining half of the book is devoted to integral equations with real symmetric kernels and to eigenfunction expansions of their kernels and solutions. Riemann integration and piecewise continuous kernels are assumed, but an appendix is given at the end of the book indicating modifications needed to handle equations with kernels that are square integrable in the Lebesgue sense. Analogies between n-dimensional Euclidean space and function spaces in which the integral equation is investigated are emphasized by parallel columns of fundamental quantities and relations. Results include the Hilbert-Schmidt Theorem and the Schmidt (eigenfunction expansion) formula for a solution.

In addition to the appendix on Lebesgue integration, a short appendix on reduction of a quadratic form to a canonical form is given. Besides an adequate index, a handy list of theorem titles and their page references is provided.

This book is a good text for an introductory course in integral equations at the advanced undergraduate or first-year graduate levels or for self-study. Although a few examples and exercises are provided, additional examples, applications and exercises should be provided if used as a text. The logical order and motivation for the theorems make for rapid understanding.

JOHN H. BARRETT

General theory of Banach algebras. By Charles E. Rickart, New York, Van Nostrand, 1960. 11+394 pp. $10.50.

A report on Soviet mathematics, written by Professor J. P. LaSalle and summarizing the result of a year's study by a panel organized by RIAS, appeared in the February 1961 issue of the Notices of this Society. Concerning functional analysis it states, in part, " . . . It is these applications to other fields of mathematics that are of greatest interest to Soviet mathematicians. By contrast, American mathematicians are primarily concerned with the structure of functional analysis and the achievement of more abstractness and greater generality. . . . Special mention should be made of the number of excellent textbooks on this subject that have recently been published . . . . In the presentation of results and the explanation of current research they are far ahead of anything the west can claim."

This important book by an American mathematician on a branch of functional analysis is a counter-example to the last statement. In the presentation of results and the explanation of current research it measures up to Russian or American or any other standards. As its title indicates, the book is definitely and deliberately written in the "American" tradition. The author's point of view, as stated in the preface, is that "It becomes increasingly evident that, in spite of the deep and continuing influence of analysis on the theory of Banach algebras, the essence of the subject as an independent discipline is to be found in its algebraic development." Accordingly he provides a systematic account of the general theory of Banach algebras emphasizing structure and representation theory. Examples and applications are by no means slighted; indeed a very interesting and valuable