BOOK REVIEWS

Introduction to Fourier analysis and generalised functions. By M. J. Lighthill. New York, Cambridge University Press, 1958. 8+78 pp., $3.50.

The theory of generalised functions (distributions) is playing an ever increasing role in modern analysis. Therefore, an elementary but rigorous account of this theory for advanced undergraduates is a welcome addition to the more advanced and comprehensive books of Laurent Schwartz [*Théorie des distributions*, vols. 1 and 2. Paris, Hermann et Cie, 1950–1951]. In addition to presenting the basic results on Fourier transforms and series via distributions, the author develops a simple, systematic method for estimating asymptotically Fourier transforms and coefficients for a wide class of functions. Unfortunately, the author gives the impression that the delicate and interesting questions of the classical theory can be dispensed with by employing distributions. Of course, this is only true to the extent that he is only concerned with questions which are more easily investigated in the framework of a theory based on distributions.

The material in this book could easily be covered in a one-semester course. Many students might find the pace a bit too strenuous without sufficient amplification since the proofs and examples are in the main only sketched. The examples are quite good but, for a text, there are too few exercises.

The book contains five short chapters. The first chapter is of an introductory nature designed to motivate the use of a generalised function approach to Fourier analysis. Starting with a brief account of the theory and applications of Fourier transforms and series, the author then emphasizes the inadequacy of the classical theory to permit many of the formal operations needed in most applications. An indication is given of the wider applicability of a theory of Fourier analysis based on distributions.

The basic theory of distributions and their Fourier transforms is the subject matter of the second chapter. Distributions are defined in terms of sequences following Temple [J. London Math. Soc. vol. 28 (1953) pp. 134–148]. More precisely, a distribution is defined as a sequence of "good functions" (i.e., infinitely differentiable functions with rapid decay at infinity) with the property that the sequence of linear functionals they define on the class of "good functions" be convergent. These correspond, roughly, to the so-called "distributions tempérées" of L. Schwartz. The definition of a distribution is simplified by omitting the condition that it be a *continuous* linear functional on the space of "good functions." This definition is sufficient for the purposes of the book, and it avoids certain technical complications which would only hinder the intended reader. The basic operations on distributions are developed in addition to results concerning the identification of ordinary functions and distributions. No knowledge of the Lebesgue integral is assumed. Much of the theory in this chapter is illustrated with the delta function and its derivatives.

Chapter three is devoted to the study of several particular distributions which occur frequently in practice. There is also a brief account of the distribution interpretation of Hadamard's "finite part" of an improper integral and Cauchy's "principal value." This chapter closes with a short table of the Fourier transforms of the previously studied distributions.

In chapter four, the author employs a reformulation of the Riemann-Lebesgue lemma to obtain a systematic method for obtaining asymptotic estimates of Fourier transforms of functions with a finite number of singularities. The most interesting and instructive examples are to be found in this and the next chapter.

The final chapter is devoted to Fourier series. There are two important and useful theorems in this chapter. First, a necessary and sufficient condition is given for a trigonometrical series to converge to a generalised periodic function. Second, that there exists a unique Fourier series representation of any generalised periodic function which converges to the function, whose coefficients can be determined, and which can be differentiated term by term any number of times. In the final section of this chapter, a method is given for determining the asymptotic behavior of the Fourier coefficients of generalised periodic functions. The main results apply to generalised periodic functions with a finite number of singularities in the period.

MILTON LEES

The mathematics of physics and chemistry. By Henry Margenau and George Moseley Murphy. 2d ed. Princeton, Van Nostrand, 1956. $7.95.

Anyone who has read stories about the South Seas is aware that there is a language called Pidgin English (actually there are several kinds) which seems at first sight to be a clumsy and inept parody of English. It has, as a matter of fact, attained wide currency in some places, and is now recognized as being a genuine language in its own right, although somewhat limited in its vocabulary. Its repulsive