Without (A), $m^2>\aleph_0$ implies $m>\aleph_0$. But without (A) we cannot prove that $m^2>n^2$ either implies or is implied by $m>n$ (p. 149).

Without (A) we can prove that the set of all infinite sequences of real numbers is of power $\mathfrak{c}$, but without (A) we are unable to prove that the set of all denumerable sets of real numbers is of power $\mathfrak{c}$ (p. 112).

The order types $\alpha=\omega\eta$ and $\beta=\omega(\eta+1)$ satisfy $\alpha^2=\beta^2$ and $\alpha\neq\beta$. But it is an open question whether there exist order types γ and δ for which $\gamma^3=\delta^3$ and $\gamma^2\neq\delta^2$ (p. 232).

Fermat's last theorem is false for order types (p. 232), and for ordinals with transfinite exponent (p. 318). Fermat numbers, i.e. ordinals of the form $2^{2^\alpha}+1$, are prime for every transfinite ordinal α (p. 339).

J. C. OXTOBY

Gitterpunkte in mehrdimensionalen Kugeln. By A. Walfisz. Monografie Matematyczne, vol. 33. Warszawa, Panstwowe Wydawnictwo Naukowe, 1957. 471 pp.

This is a beautifully written book by a leading expert in the field. Although of immense value to the specialist, it is addressed to a wider circle of readers. To quote the author's own words, "Das Studium des Buches setzt nur Kenntnisse voraus, wie sie in den üblichen Anfängervorlesungen über Analysis, Algebra und Zahlentheorie an den Hochschulen gegeben werden. Auch sind die Rechnungen über all sehr eingehend durchgeführt." Almost a third of the book is devoted to researches of the last ten years.

The book is concerned with the study of $P_k(X)$, which is the difference between the number of lattice-points in the k-dimensional hypersphere

$$y_1^2+y_2^2+\cdots+y_k^2 \leqq X \tag{1}$$

and its volume $V_k(X)$. So

$$P_k(X) = A_k(X) - V_k(X) \tag{2}$$

where $A_k(X)$ is the number of lattice-points satisfying (1). It is well-known that we have the asymptotic relation $A_k(X)\sim V_k(X)$.

Gauss observed that $P_2(X)=O(X)^{1/2}$. Sierpinski (1909) found $P_2(X)=O(X^{1/3})$, van der Corput proved the sharper result $P_2(X)=O(X^c)$ and $c<1/3$ and there have been petty improvements in the exponent in later years. It is also known (Hardy) that the exponent cannot be lowered below $1/4$; on the other hand it is considered highly probable that an exponent $1/4+\epsilon$, where $\epsilon>0$ is arbitrary, represents the truth. We proceed to sketch the contents of the book chapter by chapter.

Chapter 1 contains Estermann's (J. London Math. Soc. vol. 20 (1945) pp. 66–67) well-known short-cut to the determination of the sign in the Gaussian sum. Next the author obtains Jacobi's formula for the number of representations of a number as a sum of 4 squares. The treatment, involving infinite series, was rediscovered by Ramanujan (*On certain arithmetical functions*, Trans. Camb. Phil. Soc. vol. 22, 1916, pp. 159–184). The chapter concludes with a formula of Landau for $A_k(X)$ and the well-known "Singular series" representation of Hardy for $r_k(n)$, the number of representations of n as a sum of k squares, when $k>4$.

Chapter 2 contains elementary estimates of $P_k(X)$. H. Weyl's method of estimating exponential sums is used to obtain

$$P_4(X) = O\left(\frac{X \log X}{\log \log X}\right)$$

which is due to the author (Math. Z. vol. 26, 1927, pp. 66–88). This chapter concludes with the deep result obtained from L. K. Hua's and Vinogradoff's methods:

$$P_4(X) = O(X \log^{3/4} X (\log \log X)^{1/2}).$$

Chapter 3 contains "Ω-results" (the Ω-notation is due to Littlewood), e.g.

$$P_k(X) = \Omega(X^{k/2-1}),$$

$$P_4(X) = \Omega(X \log \log X).$$

The rest of the chapter studies the function

$$\rho_k(t) = \frac{P_k(t)}{t^{k/2-1}}$$

($\rho_k(X)$ is bounded for $k>4$ as $X\to\infty$). In particular we have the result: the sequence of numbers $\rho_k^{(1)}, \rho_k^{(2)}, \rho_k^{(3)}, \cdots$ has infinitely many limit points for $k\geqq 5$.

Chapters 4 and 5 contain results of Petersson (Hamburg Abhandlungen, vol. 5, 1926, pp. 116–150) and Lursmanaschwili-Walfisz. These are too complicated to be quoted here.

Chapter 6 and Chapter 7 have more on the functions P_k and ρ_k for even k and odd k respectively.

Chapter 8: $\int_0^X P_4^2(y)dy = 2/3\pi^2 X^3 + 0(X^{5/2})$.

This is heavy going—a formidable exercise in analytic number theory! The result is due to Walfisz.

Chapter 9: $\int_0^X P_k^2(y)dy$ (Jarnik).

Chapter 10: Development of $P_k(t)$ in Bessel function series.

The author remarks that the study of ellipsoids: $\alpha_1 y_1^2 + \alpha_2 y_2^2 + \cdots + \alpha_k y_k^2 \leq X$ (with positive irrational coefficients α_j) would require a separate monograph. The reviewer would also like to mention in this connection the striking recent work of Davenport, Heilbronn, G. L. Watson on irrational indefinite quadratic forms and that of Birch and D. J. Lewis (Mathematika, December, 1957) on the nontrivial representation of 0 by "mixed" cubic forms (with coefficients in any algebraic number field) with a sufficient number of variables.

S. CHOWLA

Lectures on ordinary differential equations. By Witold Hurewicz. The Technology Press of the Massachusetts Institute of Technology, and John Wiley, New York, 1958. 17+122 pp. $5.00.

We quote from the Preface. "This book is a reprinting, with minor revisions and one correction, of notes originally prepared by John P. Brown from the lectures given in 1943 by the late Professor Witold Hurewicz at Brown University. They were first published in mimeographed form by Brown University in 1943, and were reissued by the Mathematics Department of the Massachusetts Institute of Technology in 1956. . . . An appreciation of Witold Hurewicz by Professor Solomon Lefschetz, which first appeared in the *Bulletin of the American Mathematical Society*, is included in this book, together with a bibliography of his published works."

The book consists of five short chapters. The first presents the existence and uniqueness results for the equation $y' = f(x, y)$ with f continuous and satisfying a Lipschitz condition. The existence result under the assumption of continuity alone is proved using the Weierstrass approximation theorem and equicontinuous sequences. The dependence of solutions on initial conditions and parameters, and the continuation of solutions, are also treated here. In Chapter 2 it is indicated how all these results carry over to systems. Chapter 3 is concerned with elementary properties of linear systems, and in particular linear systems with constant coefficients. The last two chapters deal with the more geometric aspects of the subject. In Chapter 4 is discussed the behavior of solutions in the vicinity of an isolated singularity of a system $x' = P(x, y)$, $y' = Q(x, y)$ by considering it as a perturbation of a linear system. Chapter 5 is devoted to a proof of the Poincaré-Bendixson Theorem, and a short discussion of orbital stability.