to read. What were, before, carefully articulated discussions have become, somehow, amorphous. It is hard to imagine that the saving in cost is worth the loss incurred.

ARLEN BROWN

The topology of fibre bundles. By Norman Steenrod. Princeton, Princeton University Press, 1951, second printing, 1957. 8+227 pp. $5.00.

This second printing of Steenrod's well known book differs from the first only in the addition of an appendix which describes progress in the field between 1951 and 1956. The rate of progress has been very high indeed, so that even this appendix requires amendment to bring it up to date.

Today as in 1951 the term "fibre space" is ambiguous: the definitions due to Serre, Hu, and Hurewicz all have a good claim to the title. However, thanks to Steenrod's book, the "fibre bundle" is now a familiar and well defined object. In the applications of topology to differential geometry, and lately also to algebraic geometry, the fibre bundle is a tool of fundamental importance.

The following is a brief description of the book. Part I sets a foundation for the study of fibre bundles. Concepts such as cross-section, bundle map, induced bundle, and principal bundle are defined and studied with the author's characteristic thoroughness. A number of important examples are considered: tensor bundles, covering spaces, the principal bundle over a coset space, and so on. It is shown that any cross-section of a differentiable bundle can be approximated by a suitably differentiable cross-section. As is pointed out in the appendix, many of the theorems of Part I can be technically improved. However this remains the best presentation of the subject matter which is available.

Part II studies the homotopy theory of bundles. The homotopy sequence of a bundle is defined; a classification theorem for bundles over the n-sphere is proved; the theory of universal bundles is developed; and the Hopf fiberings are studied. A number of results about the homotopy groups of spheres and other standard manifolds are obtained. However this last material has been outclassed by subsequent developments in the field. [For recent work see: R. Bott, *The stable homotopy of the classical groups*, Proc. Nat. Acad. Sci. U.S.A. vol. 43 (1957) pp. 933–935; F. Adams, *On the structure and applications of the Steenrod algebra*, Comm. Math. Helv., to appear; and G. F. Paechter, *The groups* $\pi_r(V_{n,m})$ (I), Quart. J. Math. Ser. (2) vol. 7 (1956) pp. 249–268.] Finally the tangent bundle of the n-sphere is studied. [For recent work see forthcoming papers by Kervaire

(Proc. Nat. Acad. Sci. U.S.A.); by Bott and Milnor (Bull. Amer. Math. Soc. vol. 64 (1958) pp. 87–89.; and by James (Proc. London Math. Soc.).]

Part III, *The cohomology theory of bundles*, contains an excellent exposition of obstruction theory, using bundles of coefficients. This theory is applied to the study of Stiefel-Whitney classes; however in view of the remarkable developments due to Thom and Wu, this approach to Stiefel-Whitney classes has become more or less obsolete. The next section studies the problem of whether or not a given n-manifold possesses a continuous field of quadratic forms of signature k. (I.e. does the tangent bundle split into the "Whitney sum" of a k dimensional vector space bundle and an $(n-k)$-dimensional vector space bundle?) Finally the Chern classes of a complex analytic manifold are considered.

The following is an illustration of the way in which material in Steenrod's book has led to important research. The problem of classifying sphere bundles over spheres occupies only six pages of the book. This led to work by James and Whitehead on the problem of classifying the resulting total spaces as to homotopy type. [See Proc. London Math. Soc. vol. 4 (1954) and vol. 5 (1955).] Hirzebruch used S^2-bundles over S^2 to show that the same differentiable manifold may possess several distinct complex structures. [Math. Ann. vol. 124 (1951).] The reviewer used S^3-bundles over S^4 to show that the same topological manifold may possess several distinct differentiable structures. [Ann. of Math. vol. 64 (1956).] Several authors have used S^3-bundles over S^4 to show that manifolds of the same homotopy type may be distinguished by their Pontrjagin classes. [R. Thom, Ann. Institut Fourier, Grenoble vol. 6 (1955–1956) p. 81; I. Tamura, J. Math. Soc. Japan vol. 9 (1957); and N. Shimada, Nagoya Math. J. vol. 12 (1957).]

In conclusion, this book remains a must for students of topology and geometry.

JOHN MILNOR

Differential equations: *Geometric theory* by S. Lefschetz. New York, Interscience Publishers, 1957. 10+364 pp. $9.50.

The present work is modestly referred to by the author as an extension of his previous Annals of Mathematics Study, *Lectures on differential equations*. Actually, the additional material included makes this a book which differs in character from its predecessor. The *Lectures on differential equations* was strictly a textbook for, say a first year graduate course in ordinary differential equations. The present volume, while it contains some introductory material (Chap-