

(b) Necessary and sufficient conditions for mean convergence are mentioned only in an exercise (§5, Exercise 22) and even there the treatment leaves something to be desired. It is not pointed out that the conditions are necessary, and it is not pointed out that the three conditions can be reduced to two.

More important than these and other detailed objections is the fact that the authors still conceal the connection between measure theory and set theory. As in Chapters I–IV, measures are still pictured almost exclusively as linear functionals on spaces of continuous functions. Regardless of the merits of this approach to the subject, to raise a student on a diet of that and nothing else will probably retard his ability to communicate with the outside world. And, extol the virtues of Bourbaki as you will, there still is an outside world.

The reviewer feels very strongly that textbook authors, be they Bourbaki or mere mortals, should realize that their task is to supplement the literature, not to supplant it.

M. E. MUNROE

Generators and relations for discrete groups. By H. S. M. Coxeter and W. O. J. Moser. *Ergebnisse der Mathematik und Ihrer Grenzgebiete, New Series*, no. 14. Berlin-Göttingen-Heidelberg, Springer, 1957. 8+155 pp. DM32.

It is refreshing to find a book that not only studies groups but also deals with many particular and interesting groups. The major theorems of group theory have substance only insofar as they apply to actual groups. The Mathematician with any feeling for groups will welcome this monograph and its rich display of groups of many kinds.

The monograph deals for the most part with finitely presented groups, that is groups G generated by a finite number of elements R_1, R_2, \dots, R_m subject to a finite number of defining relations $g_k(R_1, \dots, R_m) = 1, k = 1, \dots, s$. There are two main faces to the study of finitely presented groups. The obverse is the problem of studying the properties of a group defined by given relations. Among other things we wish to know if the group is finite and if so, what its order is. The reverse is the problem of finding a simple set of defining relations for a given group. Both these problems are studied in this monograph, and a variety of methods, mostly geometrical, are employed. Since the word problem for groups is unsolvable, we are relieved of the necessity of searching for an all embracing method and may enjoy the elegance of several diverse approaches.

A practical method of enumerating cosets is given. This method has the advantage that on completion it yields a permutation repre-

sentation of the group. A typical application of this method consists in showing that if G is defined by

$$V_1^3 = V_2^3 = V_3^3 = (V_2V_3)^2 = (V_3V_1)^2 = (V_1V_2)^2 = 1$$

then the subgroup generated by V_1 and V_2 is of index 5.

A large part of the monograph is geometric in its orientation. Chapters 3, 4, and 5 deal with graphs, Cayley diagrams, crystallography, and fundamental groups of certain manifolds. The Cayley diagram of a group G is a graph with one vertex for each element g of G and a directed arc associated with a generator x joining the vertex g to gx , there being an arc for every element g and generator x . The fundamental groups of two dimensional manifolds are studied. The seventeen two dimensional space groups are given in full.

Groups with special kinds of defining relations are discussed at length. Thus the group defined by $R^h = S^m T^n = RST = (TRS)^q = 1$ is designated as (h, m, n, q) . Fortunately a series of tables in the appendix lists these notations and the selections in which they appear.

The regular tessellations of a space are associated with a group. The group is finite if the space is elliptic, but is infinite if the space is Euclidean or hyperbolic. Chapter 5 is devoted to this subject.

There is a brief discussion of Burnside's problem. In this connection I wish to correct a statement attributed to me. The Burnside group of exponent 4 with two generators is of order 2^{12} and has the defining relations

$$\begin{aligned} A^4 = B^4 = (AB)^4 = (A^{-1}B)^4 = (A^2B)^4 = (AB^2)^4 \\ = (A^2B^2)^4 = (A^{-1}B^{-1}AB)^4 = (A^{-1}BAB)^4 = 1. \end{aligned}$$

My statement was that the relation $(A^{-1}B^{-1}AB)^4 = 1$ is superfluous, not that $(A^{-1}BAB)^4 = 1$ is superfluous.

A great deal of space is devoted to finding simple defining relations for a number of known groups. First the symmetric and alternating groups are treated, there being an extensive literature on this subject. The connections with braid groups are noted and discussed. An entire chapter is devoted to defining relations for the unimodular and linear fractional groups over finite fields with the Mathieu group on 11 letters thrown in for good measure.

The last two chapters are purely geometric in their approach. Chapter 8 treats the regular maps on multiply-connected surfaces, this investigation being similar to the study of regular tessellations except that the conditions are now topological rather than metric. Chapter 9 *Groups generated by reflections* includes a vast number of

groups, in particular the Dickson-Chevalley finite analogues of the exceptional simple Lie groups.

MARSHALL HALL, JR.

Games and decisions: Introduction and critical survey. By R. Duncan Luce and Howard Raiffa. John Wiley and Sons, Inc., 1957. 19+509 pp. \$8.75.

This is a book written by mathematicians but "aimed primarily at those readers working in the behavioral sciences." It "attempts to communicate the central ideas and results of game theory and related decision making models unencumbered by their technical mathematical details: thus, for example almost no proofs are included." Despite these remarks from the authors' introduction, the book deserves the attention of professional mathematicians, behavioral scientists or anyone else interested in finding out about the subject matter of its title. Of these various groups I strongly suspect it is the mathematicians who will most readily understand its contents.

The book is unique in many respects. It is the first on the subject which attempts to cover the whole field, a feat it succeeds in doing with almost astonishing comprehensiveness. The authors have read, digested and present here in a lucid manner virtually every idea on games and decisions that has been put forth since these objects became the subject matter of a "theory." Even the game theory specialist will in all likelihood find branches treated here with which he is not familiar (this reviewer is grateful to the authors for the sections on statistical decision making).

The material of the book is divided up according to the following scheme: a general introductory chapter on games, a chapter on utility theory and then another more technical chapter on games, describing the extensive and normal forms. There follow three chapters on two-person games, the first on the, dare we call it, "classical" zero-sum theory, the next two on nonzero sum games, first the non-cooperative theory, then the cooperative. Already we note a sharp contrast with other books which have appeared since von Neumann and Morgenstern, and which have devoted all or almost all their attention to the zero-sum theory. The present authors hasten on to more unsettled and controversial parts of the theory which constitute their main object of study. Nevertheless, for the sake of completeness, the book includes no less than seven appendices to the two-person zero-sum chapter, covering such related topics as linear programming, infinite and sequential games among others. The next five chapters are devoted to n -person games and a presentation of