erties which are likely to be of more importance with regard to applications than others are treated in more detail. Among the contributions since the appearance of Watson's book, mainly those with connection to applications are included. This of course is quite consistent with the purpose of this book. However, it seems regrettable to the reviewer that Chapter 5 (*Asymptotic expansions*) does not contain anything of the work done by R. E. Langer, T. M. Cherry, F. W. Olver and others. Likewise regrettable is the absence of a more detailed and organized bibliography. But these points may be of minor concern for the majority of the users of this book.

F. OBERHETTINGER

Symposium on Monte Carlo methods. Held at the University of Florida, March 16–17, 1954. Edited by H. A. Meyer. New York, Wiley, 1956. 16+382 pp. $7.50.

The term Monte Carlo methods was coined during the second World War to describe the use of random sampling for the numerical solution of mathematical problems. The questions that gave rise to these techniques had to do with the penetration of radiation into matter, in particular with the shielding properties important in the technology of atomic energy. Instead of solving the complicated integro-differential equations describing these processes by analytical or orthodox numerical methods it was proposed to simulate the physical phenomenon on computing machines by constructing artificially a sufficiently large number of particle biographies. Thus, in dealing with a process in which an elementary particle is subject to collisions resulting in absorption or reflection, possibly with a partial loss of energy, a moving particle with given initial energy and direction would be assumed and its subsequent history decided by playing a sequence of games of chance. The outcome of these games would decide from step to step the free path lengths, the nature of the collisions, the angles of reflection, the loss of energy in the collision, etc. The probability distributions underlying each game must, of course, be taken from physical theory. If a sufficient number of such fictitious biographies have been calculated, physically relevant questions such as the amount of radiation penetrating a given slab can be answered with reasonable accuracy.

The idea of solving mathematical problems by statistical experimentation is not new. Buffon's needle experiment for the determination of π is well known, and statisticians have occasionally used such procedures to obtain preliminary information on the shape of distribution functions under mathematical investigation. However, these new applications were more interesting, since they were farther removed from questions in statistics proper and at the same time numerically useful. Also, the picturesque name given to the method helped to arouse the interest of numerous mathematicians. The advent of high speed computing machines made it likely that this procedure would have a bright future, since it requires that a fairly simple algorithm be repeated a large number of times, something for which fast machines are excellently suited.

From the purely mathematical point of view it is a challenging question to give a probabilistic interpretation to problems which did not originate from a physical situation involving probability considerations. The classical example for this is the finite difference approximation to Dirichlet's problem based on the equation

$$U(x + h, y) + U(x - h, y) + U(x, y + h) + U(x, y - h) - 4U(x, y) = 0,$$

which has a well known simple interpretation in terms of random walks in a grid in the presence of an absorbing boundary. Many other differential and integral equations, as well as quadratures and problems in linear algebra can also be connected with problems in probability.

It soon turned out that even with the fastest machines available simple straightforward sampling was frequently inadequate. This is a consequence of the fact that the accuracy of the results increases only as $n^{1/2}$ with the sample size n. Therefore a considerable literature has sprung up devoted to the improvement of the method by means of more refined sampling procedures such as quota, stratified, importance sampling etc., some of which had long been used by statisticians.

Another problem that had to be confronted was the generation of random sequences of numbers. In order to make full use of the speed of modern computing machines it is desirable that these sequences be generated in the machines as the computation proceeds. Also, it is important for the purpose of checking the calculations, that such a sequence be reproducible at will. It is therefore agreed now that the best way to supply such random sequences is to use some number theoretical process which generates sequences of numbers that satisfy the common criteria of randomness. There are some philosophical questions inherent in the simulation of a chance process by a strictly deterministic procedure. These misgivings are brought out well in D. H. Lehmer's definition of such pseudo-random sequences as "A vague notion embodying the idea of a sequence in which each term is unpredictable to the uninitiated and whose digits pass a certain number of tests, traditional with statisticians and depending somewhat on the uses to which the sequence is to be put." But these doubts do not seem to affect the practical usefulness of such methods.

If the numbers of the random sequence employed have a uniform probability distribution while the problem at hand requires some other distribution a problem of conversion arises which turns out to be less trivial than one might first think. This is another line of research on which some effort has been spent.

The volume reviewed here gives an excellent survey of the present stand of the Monte Carlo method. It may not be unfair to apply a statistical analysis to a collection of papers on this subject. The table below lists the number of articles devoted to the various aspects of the theory.

Generating and testing of random numbers	4
Sampling methods	4
Diffusion, reflection and scattering problems	4
Statistical problems	2
Linear algebra	2
Numerical quadratures	2
Partial differential equations	2
Integral equations	2
Game theory	1

(Some articles fall into more than one category. Therefore the total is 23, while there are only 20 papers in the book.)

This list shows that the "how" is rather heavily emphasized by comparison with the "what." This impression is strengthened if it is noted that only four papers contain genuine numerical applications other than illustrative examples. This reviewer cannot be sure whether this bias reflects accurately the ratio of theory and practice in current work on or with Monte Carlo methods, or if there exists a large body of useful routine applications which do not warrant publication as a research paper in such a volume, but he suspects that the first interpretation is correct.

The book contains an extremely useful bibliography with abstracts of about 125 articles on Monte Carlo proper and a list, mostly without abstracts, of about 250 papers and books on related topics.

A breakdown of this material gives the impression that by far the most important field of application of Monte Carlo techniques is still the original one of particle diffusion. Problems not intrinsically connected with probability have given rise to a considerable theoretical literature, but since no worth while computational application has been forthcoming, the interest has flagged in recent years, perhaps only temporarily.

There is no space here to report extensively on the contents of the individual articles. This reader found the paper by H. Kahn a very readable introduction to Monte Carlo sampling techniques. The paper by O. Taussky and J. Todd contains a particularly useful exposition of the generation of random sequences. H. F. Trotter and J. W. Tukey have contributed an article on "Conditional Monte Carlo for normal samples" which contains some very clever ideas by means of which the sample size necessary for satisfactory accuracy is reduced by the factor 5000 in a statistical application reported by H. J. Arnold, B. D. Bucher, H. F. Trotter and J. W. Tukey. The first of these two papers is rather difficult to read, partly because its authors adopt the extremely colloquial style frequently met in work on Monte Carlo. The results of J. H. Curtiss concerning the relative efficiency of Monte Carlo procedures and ordinary numerical methods for the solution of systems of linear algebraic equations raises the hope that there is still a future for Monte Carlo in this particular field.

This volume is almost indispensable to mathematicians doing research on or with Monte Carlo methods, and it can be highly recommended to readers who wish to find out what the Monte Carlo method really is.

WOLFGANG WASOW

Automata studies. Edited by C. E. Shannon and J. McCarthy, Princeton, New Jersey, Princeton University Press, 1956, viii +285 pp. $4.00.

This collection of essays is divided into three sections: "Finite Automata," "Turing Machines," and "Synthesis of Automata." We will discuss the first two sections together and take up the third later.

The essays in these two sections treat mainly the mathematical and logical theory of quantized or discrete automata, as contrasted with analog machines. The automata of the first section have a fixed number of elements and states, while those of the second section have a changing, but always finite, number of states. An equally basic division is into *deterministic* and *probabilistic* automata, according to whether the state of an automaton is a deterministic or probabilistic function of the preceding state (including the inputs); this classification crosses the one used by the editors (e.g., there are two kinds of deterministic automata, fixed automata and Turing machines). Since realizability by a deterministic automaton is equivalent to (partial)