

Elementary topology. By D. W. Hall and G. L. Spencer, II. New York, Wiley, 1955. 12+303 pp. \$7.00.

The textbook reviewed here is meant for an undergraduate course (possibly even for juniors) with this dual objective: first, to produce such understanding of the facts and techniques of elementary topology as "will help the student immeasurably in his courses in advanced calculus, real variables and complex variables . . ."; and second, to constitute that course in which the student is definitely acquainted with rigor. It does present the basic material carefully and patiently, with many exercises, a good part of which requires much reexamination of the proofs given; and there are twenty-six figures. Using the book in the way suggested would involve (in most schools) serious revision of the program of courses. The book ought also to be considered simply as an introduction to topology.

The following topics are included in the first four and one-half chapters, which are suggested, on the basis of experience, for an undergraduate year course: (1) Introductory set theory; (2) The real number system; (3) Concepts such as metric, closure, compactness, etc.; (4) Metric spaces more intently, including metrics specially related to local connectedness, completion, the space (C) and its completeness, Urysohn's metrization theorem, etc. Among the exercises we find Directed sets, the Hilbert cube and Baire's density theorem (not involved in the regular text) suggested with references as topics for a series of short papers at this point; (5) The study and characterization of arcs, curves, the 1-, and 2-sphere, and (other) spaces which are continuous images of the unit interval, to the extent of fifty pages, which is nearly half of the fifth chapter. This chapter is the real core of the book. In a graduate course it could probably be covered together with the remaining two: (7) Partitionable spaces (R. H. Bing's fruitful concept); (8) The axiom of choice, containing also infinite topological products (that is, those with more than a finite number of factors) and Tychonoff's theorem.

In my estimation this book would be very useful in a graduate semester course in set theoretic topology where it is desired to include the carefully prepared material of the fifth chapter. The authors deserve praise for exhibiting a teeming arena for the application of easily intelligible topological techniques, to say nothing of the inherent interest of the topics already mentioned (and Jordan's curve theorem, to mention another).

They contend that the treatment has been kept especially elementary through the careful avoidance of algebraic arguments. Indeed,

no groups of any sort are introduced. It thus opposes the widely held opinion that algebra can clarify and simplify topology. However that may be, modern topology does involve a lot of algebra; and if indeed there is anything tricky about the application of algebra to topology, then the sooner its utility is gently illustrated to the student the better. There is, of course, considerable topological activity concerned with phenomena in lower-dimensional Euclidean spaces in which it appears possible to do quite well without some of the tools being sharpened in algebraic topology; and for such research this book will prepare the student quite well.

If the needs of the future algebraic topologist were thus as a matter of pedagogic (not sectarian) policy left to one side, the needs of the future topologic algebraist were overlooked altogether. Uniform structures are presented only with a countability restriction; function spaces are treated almost not at all. Infinite topological products are presented merely as an application of the axiom of choice, and Tychonoff's theorem appears as a variant of this axiom. (The treatment of the axiom of choice is marred by a preoccupation with what seems to be an unnecessary axiom of finite choice.) Metrizable product spaces is not discussed (and therefore the Hilbert cube has to be carefully circumvented in the earlier section of metrization). One would have expected the theorem on the connectedness of product spaces. These defects can easily be repaired by a competent instructor, and are thus minor. We invite anyone preparing a course in topology to consider this work on the merit of the substance and treatment of the fifth chapter which is even physically half the book.

RICHARD ARENS

Plane waves and spherical means applied to partial differential equations. By Fritz John. New York, Interscience Publishers, Inc., 1955. 8+172 pp. \$4.50.

This book is dedicated, as the title indicates, to an exposition of the author's results on various problems concerning partial differential equations, results obtained by using relations between a function in an n -dimensional Euclidean space and its integrals over planes and spheres. While treatises and encyclopedia articles supposedly strive for completeness either in the extent of subject-matter treated or in its detailed presentation, a tract has no such pretension and may present a slice of the subject-matter of its field organized to the author's taste. The present example, the second of a new series of Interscience Tracts, has been fitted to a very elegant taste indeed. Yet while concentrating on techniques and ideas which he has personally developed and finds congenial, Fritz John has touched upon