The topic of elliptic equations occupies the third chapter. After a short proof, due to Privalov, of the minimum-maximum theorem for solutions of the Laplace equation, the author turns to the Dirichlet problem for the circle. Using the Poisson integral representation, the standard sequence of theorems concerning harmonic functions is derived. The existence of a solution for general regions is then proved using the Poincaré-Perron concept of superharmonic functions. In order, the author discusses the exterior boundary value problem, the Neumann problem, potential theory, and the application of potential theory to the solution of boundary value problems. Following these results, there is a brief discussion of the approximate solution of the Dirichlet problem by means of finite differences, following an approach due to Lusternik. As in the previous chapters, this chapter closes with a survey of some of the most important results for elliptic equations.

The last chapter is a very brief one, and sketches the application of some of the techniques developed in the previous chapters to the solution of some simple problems in the theory of heat conduction.

The only fault in the book is a small one—there is no index. The bibliography is given in footnotes, and this also is not completely desirable. Apart from these minor items, the book is highly to be recommended. It is printed very attractively, reads very smoothly, and all in all is to be regarded as an elegant introduction to an attractive field of mathematics.

RICHARD BELLMAN

Vorlesungen über Approximationstheorie. By N. I. Achieser. Berlin, Akademie-Verlag, 1953. 10+309 pp., 10 figures. 29.00 DM.

This splendid book (translated from the Russian edition of 1947) gives much more than the title promises. Besides a discussion of specific problems of approximation it provides also an introduction to many different parts of analysis, as can be seen from the following list of topics in Chapter I: Elements of functional analysis. Chapter II: Approximation in C (Chebyshev approximation). Chapter III: Fourier analysis: L^2-theory, Fejér's theorem, Watson transforms, conjugate functions. Chapter IV: Entire functions of exponential type bounded on the real axis, S. Bernstein's inequalities for the derivatives of these functions and generalizations. Chapter V: Problems of best approximation by trigonometric polynomials and by entire functions of exponential type. Chapter VI: Wiener's Tauberian theorem.

The presentation is very clear and interesting. The author has achieved a happy mixture of the abstract and the concrete. Definitions are immediately followed by examples and general theorems are strikingly applied to special cases. For example, the definition of the modulus of continuity is illustrated by a computation of the modulus of continuity of Weierstrass' nondifferentiable function. The judicious blending of modern and classical methods also appears in the proofs. In particular, the chapter on Wiener's theorem is probably the easiest account of the subject available in print, although the motivation for the notions used (normed ring, ideal) may be obscure to the beginner.

The "general problem of approximation" is formulated as follows by the author: $f(x)$ and $F(x; A)$ are two functions of a normed function-space; the second of these depends on a number of parameters A. It is required to choose the values of the parameters in such a way that $\|f-F\|$ becomes a minimum. The most important case is the linear case, $F(x; A)=F(x; a_1, a_2, \cdots, a_n)=\sum_{k=1}^{n} a_k f_k$ where $\{f_k\}$ $(k=1, 2, \cdots)$ is a given set of functions. The book contains concise proofs of the classical results of S. Bernstein, Chebyshev, Haar, D. Jackson and de la Vallée Poussin concerning the existence and uniqueness of the extremal functions for various choices of $\{f_k\}$ in the spaces L^p and C. The problem of the closeness of the approximation which can be achieved also receives careful attention. Chapters IV and V will be of particular interest to Western readers, since they contain an account of fairly recent Russian investigations. A large part of Chapter IV is concerned with generalizations of Bernstein's theorem: If $f(z)$ is an entire function of order 1, type σ, bounded by M on the real axis, then $|f'(x)| \leqq \sigma M$ (x real). The ideas underlying this work are the same as those used by R. C. Buck (see R. P. Boas, *Entire functions*). As an example of other work in this chapter I quote Krein's theorem: $\lambda(t)=\int_{-\infty}^{\infty} e^{ixt}d\omega(x)$ $(|t| \leqq \sigma)$, $\int_{-\infty}^{\infty}|d\omega| \leqq M$ is equivalent to $|\sum_{k=1}^{n} c_k\lambda(t_k)| \leqq M \sup_x |\sum_{k=1}^{n} c_k e^{it_kx}|$ for all integers n, all sequences of complex numbers $\{c_k\}$ and all choices of the t_k in $-\sigma \leqq t \leqq \sigma$. Chapter V is concerned with approximation by functions in $G=G(T)$, the class of all functions which are restrictions to the real axis of entire functions of order 1, type $\sigma<T$. For several important classes K of functions it is possible to determine explicitly the "best approximation" $\sup_{f\in K} \inf_{g\in G} \|f-g\|$, where the norm is either the C-norm or the L^p-norm $(p \geqq 1)$. Jackson's theorem, "If $f(x)$ has period 2π and $|f^{(r)}(x)| \leqq 1$, then there is a trigonometric polynomial $T(x)$ of order $<n$ such that $|f(x)-T(x)| \leqq A_r n^{-r}$," can be deduced from these results together with the best possible value of the constant A_r. There is an appendix which contains interesting special results, most of them concerning closure and best approxima-

tion by polynomials. An example is the following theorem. Let $\sigma(t)$ be a nondecreasing function, $\sigma'(t)$ the derivative of its absolutely continuous part. Let $L^p(d\sigma)$ ($p \geqq 1$) be the space of functions $f(t)$ ($0 \leqq t < 2\pi$) with $\|f\|^p = \int_0^{2\pi} |f(t)|^p d\sigma(t)$. The set $\{1, e^{it}, e^{2it}, \cdots\}$ is total in $L^p(d\sigma)$ if and only if $\int_0^{2\pi} | \log \sigma'(t)| dt = \infty$.

The book is handsomely printed, but the list of typographical errors at the end is far from complete. In particular, the reader should be encouraged to read p. 233 before p. 232.

W. H. J. FUCHS

BRIEF MENTION

Equazioni differenziali. By F. G. Tricomi. 2d ed. Torino, Einaudi, 1953. 353 pp. 4000 lire.

The revised edition of this book follows very closely the pattern of the first edition, which was reviewed in this Bulletin vol. 56 (1950) pp. 195–196. The most important cases of inclusion of new material are: (i) Chapter II has been augmented by an introduction to the subject of relaxation oscillations; (ii) Chapter IV has been revised considerably, to provide a more comprehensive treatment of the asymptotic character of solutions of differential equations of the form $y'' + Q(x)y = 0$.

Details of discussion have been altered in various instances, notably in Chapter IV in the treatment of the polynomials of Laguerre and Legendre. Material on the "method of Fubini" that formed an Appendix in the initial edition has been incorporated in Chapter IV; also, a number of new references have been added to the bibliography.

In this new edition the author has produced a commendable improvement of the highly interesting and valuable first edition.

W. T. REID

Linear operators. Spectral theory and some other applications. By R. G. Cooke. London, Macmillan, 1953. 12+454 pp. $10.00.

This book contains an introductory chapter, and then a chapter on quantum mechanics. These are followed by a long section (three chapters) devoted to various proofs of the spectral theorem. For a textbook treatment of this, the reviewer prefers the snappy handling in [1]. A sixth chapter is concerned with "projective convergence and limit in matrix spaces and rings." Chapter 7, the final chapter, is a self-contained exposition of the elements of the theory of Banach algebras. The theory of Banach algebras without unit element is included (i.e. the theory involving adjunction of a unit element, with