$$\lambda(\sigma) = 2 + A(1 - \sigma)^a \qquad (1 - b \leqq \sigma \leqq 1),$$

where A, a, b are positive constants (e.g., $A = 600$, $a = 1/100$). He also states that, if (7) is replaced by a similar hypothesis with a fixed α in $1/2 < \alpha < 1$, then (6) holds with

$$\lambda(\sigma) = 2 + 3\epsilon^{1/9} \qquad (\alpha + \epsilon^{1/3} \leqq \sigma \leqq 1).$$

But the proof is unfortunately omitted. (To reconcile the contradictory announcements in lines 1 and 22 on p. 162, read XXXVIII for XXXIX in line 22.)

In work of this kind it is perhaps inevitable that the central idea should sometimes be submerged by details. To reduce this danger the author has relegated some of the incidentals to six Appendices. Something more might be done in the same cause by a systematic use of integrals instead of sums. Thus, it would be possible to reduce to a few lines the elaborate argument used to estimate the difference on the left-hand side of (5), by writing this difference as

$$-\frac{1}{h!}\int_{\xi}^{\infty} \Delta(x) d_x\left(\frac{1}{x^s} \log^h \frac{x}{\xi}\right),$$

and using the estimate $\Delta(x) = O(xe^{-H(\xi)})$ $(x \geqq \xi)$.

In commending this stimulating book to the notice of analysts, and particularly to those interested in the analytical theory of numbers, the reviewer may perhaps be allowed two final comments on small points of wording. On pp. 131–132 it is stated that a proof of the "quasi-Riemannian hypothesis" $(\Theta < 1)$ would entail essentially all the consequences of a proof of the Riemann hypothesis $(\Theta = 1/2)$. This applies, of course, only to a restricted class of "consequences"; thus, the order of the error in the prime number theorem would be affected by one pair of zeros $\beta \pm i\gamma$ with $\beta > 1/2$. On p. 141 reference is made to the method created by Vinogradov for the "solution of Goldbach's problem." This is a momentary lapse from more accurate descriptions elsewhere, but, since this form of words has become current in the literature, it seems opportune to point out that Goldbach's problem has not been solved. It is no disparagement of Vinogradov's outstanding achievement with the sum of three primes to state that the basic problem of two primes remains a major challenge.

A. E. INGHAM

The foundation of statistics. By L. J. Savage. New York, Wiley, 1954. 16+2+294 pp. $6.00.

This book is an exposition of some of the leading ideas and techniques in modern statistics. They are supposed to hang together under the awning of *personal probability*,[1] which theory is expounded in the first four, and in a ramified way in the first eight chapters. This *personalistic* view of probability, as distinguished from the *objectivistic* (repeated events) and the *necessary* (logical implication) view, is an old one and holds that probability measures the degree of confidence a particular person has in the truth of a particular proposition. The present account, couched in *behavioristic* rather than *verbalistic* terms, is derived from the work of Bruno de Finetti. Its main outline may conveniently be gleaned from the end papers, at either end of the book. Briefly stated it is a theory of rational *decision* in a *grand world* (or a *small world*) based on a personal *preference* among *acts* in view of their *consequences* resulting from the *true state* of the world. Mathematically, an act is a function from state to consequence and it is simply ordered by the relation preference. This ordering of acts carries with it a natural ordering of sets of states or *events*. In the beginning there are inequalities; but if the ordering is *fine* and *tight* then these inequalities squeeze so hard on each other that there emerges a finitely additive *probability measure* on the events which *agrees* with the *qualitative probability*, thus making it *quantitative*, and we have something recognizable as an ordinary probability. (For the mathematician a rough picture would be a sort of Archimedean arithmetic without all the real numbers.) It is said that some subjectivists do not care for this strict quantification, as it tends to make life too easy and takes the show away from non-numerical probability. Such persons are in for another disillusionment, for just as probability measure arithmetizes the comparison of events, *utility*, a concept due to von Neumann and Morgenstern and discussed with some detail in Chapter 5, arithmetizes the comparison of acts. Now the acts are simply ordered according to the mathematical expectations of their utilities, and why not indeed, as they say it in French, confound the act with its utility? It thus turns out that an act is a random variable, and the recommended decision is to supremize its mathematical expectation. The rest is easier sailing, at least on the verbal level. Chapter 6 discusses how an *observation* affects the decision by conditioning the personal (I almost said *a priori*) probabilities. There is a good illustrative example there. Chapter 7, entitled "partition problems," treats the classical case where the decision is solely determined by an unobservable random variable or rather its distribution, which in the finitely-valued case reduces to a *partition*. This

[1] Italicized terms refer to those used in the book.

chapter contains a discussion of sufficient statistic, likelihood ratios, and sequential analysis. Chapters 9 to 14 are all concerned with Wald's decision theory, here called the *minimax theory*. It is first explained without reference to the personalistic theory, and then reinterpreted with it. The parallelism between the minimax theory and the theory of zero-sum two-person games is noted; von Neumann's fundamental theorem is stated without proof, and some recent work is mentioned. Various objections to the minimax rule are discussed in Chapter 13. Chapter 15 treats point estimation. As many as nine criteria for an estimate are discussed. A common defense for these criteria which seems to have escaped attention is that they lead to some sort of mathematics. It might be somewhat frivolous to say with Landau that they have also led to dissertations,[2] but surely it is not too much to suppose that even a hard-headed statistician may play around a little with a formula or a theorem without regard for the grand world? Chapter 16 is on testing and Chapter 17 on interval estimation for which the author seems to have little use. The curtain falls quickly on the mention of fiducial probability. There are two appendixes on expected value and convex functions. A third appendix is on bibliographic material and lists about 170 items with some comments.

Following the author's good example I shall allow myself a few personal observations. A book like this is necessarily part philosophy, and one who is not philosophically bent, as Mr. Savage clearly is, is often hard put to tell between what is critical thinking and what is quibbling about words. To such a person a good part of the discussion of the foundations of probability is typified by the following two examples. 1. Re probability: when a coin is tossed there is besides head and tail the possibility of the coin's standing on its edge or disappearing into a crevice. (For a variation on this theme see p. 15 on whether a rotten egg spoils an omelet.) 2. Re utility: some people gamble for a monetary loss in order to kill time or to cultivate good relations. (For a variation see p. 101 on the show-off flier.) I do not know how to draw a line between such bull-session stunts and more serious argumentation, but I should add that on the whole the author shows restraint in exercising his verbal felicity. As to clever discourse in general on the foundations of probability, I am in agreement with someone (whom I forget) that Poincaré has said most of the brilliant things, and what is left has been said by Borel. Indeed, it is a little surprising that Borel's book *Valeur pratique et philosophie des probabilités* (the last volume of his well-known collection), which also ex-

[2] This view was expressed to me by P. L. Hsu nine years ago.

pounds a theory of subjective probability based on betting behavior (cf. p. 60 of the book under review), is not referred to at all.

The author writes extremely well and obviously enjoys writing. To the very conservative his florid personal style may be at times disconcerting, but seeing that many mathematicians are such careless writers I think his literariness is to be commended. The adult reader may ignore his advice on doing all the exercises etc., he may even gloss over the nuances of personalisticism, but if he can overcome the initial barrage of a new terminology[3] he will enjoy most of what the author has to say and the way he says it.

K. L. CHUNG

Tables of integral transforms. Vol. II. Prepared under the direction of A. Erdélyi, New York, McGraw-Hill, 1954. 16+451 pp. $8.00.

This volume is divided into two parts of somewhat different character. The first, Chapters VIII through XV, follows the same organization plan as Volume I. [For a review thereof see this Bulletin vol. 60 (1954) pp. 491–493.] That is, the integrals are classified as transforms under the following types:

Hankel	$\int_0^\infty f(x)J_\nu(xy)(xy)^{1/2}dx$
Y-transform	$\int_0^\infty f(x)Y_\nu(xy)(xy)^{1/2}dx$
K-transform	$\int_0^\infty f(x)K_\nu(xy)(xy)^{1/2}dx$
H-transform	$\int_0^\infty f(x)H_\nu(xy)(xy)^{1/2}dx$
Kontorovich-Lebedev	$\int_0^\infty f(x)K_{ix}(y)dx$
Fractional integrals	$\int_0^y f(x)(y-x)^{r-1}dx$
Stieltjes	$\int_0^\infty \frac{f(x)}{(x+y)^r}dx$
Hilbert	$\int_{-\infty}^\infty \frac{f(x)}{x-y}dx.$

[3] It is hoped that the brief recapitulation given in the first paragraph of this review may be of some help there.