gruences, the Kloosterman sums and with various aspects of congruences.

Chapter V, *Congruences of the second degree*, contains the standard material on the Legendre and Jacobi symbols as well as the solution of the congruence $x^2 \equiv a \pmod{m}$. Among the unusual problems, one finds many on the sum of various types of Legendre symbols, the Gaussian sum $\sum_{x=1}^{m} e^{2\pi i a x^2/m}$ as well as other exponential sums.

Chapter VI, *Primitive roots and indices*, is concerned with the determination of all moduli having primitive roots and with the corresponding theory of indices. For such moduli, the congruence $x^n \equiv a \pmod{m}$ is treated. The modulus 2^α, which has no primitive root if $\alpha > 2$, is considered and the essentially unique representation $a \equiv (-1)^\gamma 5^{\gamma_0} \pmod{2^\alpha}$ is obtained when $2 \nmid a$. A large number of the problems deal with characters, character sums, and exponential sums including the Kloosterman sums.

The translation reads smoothly except for a number of exceptions, and the typography is an improvement over the original even though the right-hand margins of the pages have not been rectified. Although the errors mentioned in an insert to the sixth edition have been corrected, a considerable number have been introduced and some others have not been caught. For example, there are a number of places where the numeral 1 has been used in place of the letter l (pp. 35, 125, 133). In other places, lower case letters have been used in place of upper case letters (p. 87) or vice versa (p. 34). In at least three places (pp. 128, 128, 214), the sign $=$ is used in place of the correct sign $\leqq$. There are other errors as well. Also, unfortunately, the chapter and paragraph titles have been omitted from the tops of pages. It is to be hoped that the publisher will correct these deficiencies in a subsequent printing.

LOWELL SCHOENFELD

Enzyklopädie der mathematischen Wissenschaften. Vol. I, *Algebra und Zahlentheorie.* Part 1: A. *Grundlagen*; B. *Algebra.* 2d ed. Vol. I_1, no. 3, part I: I_1, 6, *Lineare Algebra*; I_1, 7, *Normalformen von Matrizen.* By G. Pickert. Leipzig, Teubner, 1953. 72 pp. 7.50 DM.

The new edition of the *Enzyklopädie*, which was long delayed by the war, seems now to be getting well under way, at least for the first parts, covering foundations, algebra and number theory. While retaining essentially as their goal the same one as the first edition, namely to give as complete as possible a description of the mathematics of their day, the editors have wisely dropped all the historical material which can be found in the first edition (and would have been needlessly duplicated); nor do they insist on complete coverage of the literature, and this is also a sensible move if it enables them to leave the main results in a better perspective. Finally, the general plan of the work has been deeply and drastically altered, to fit better the changes in outlook which have occured in mathematics during the last half-century; whether the editors have entirely succeeded in this respect is a question to which we shall return a little later.

At any rate nowhere is this change more apparent than in the first half of the present volume, which gives due emphasis to modern linear and multilinear algebra, a topic barely mentioned in the old *Enzyklopädie.* In 43 dense pages, it gives a luminous and thorough description of the concepts and results of this theory. The author has here followed rather closely the arrangement, notations and terminology of Bourbaki's corresponding chapters; modules, vector spaces, duality, linear equations are treated in this order; matrices duly come *after* linear transformations. The notion of tensor product is next taken up, in relation to multilinear mappings, and is followed by exterior algebra, which naturally leads to determinants, and their relations to duality in exterior algebra. In addition, much valuable material, not to be found in Bourbaki's treatise, is included here, such as a rather complete discussion of the abstract notion of linear dependence, various results on bases of modules over rings satisfying some kind of chain condition, determinants over sfields, the Kronecker identities for minors of a symmetric determinant (where Lepage's proof, which is only mentioned in a footnote, would have deserved a more explicit discussion), and several other interesting identities taken from Grassmann's works. But the author manages to bring in all this detail, and to give an almost exhaustive bibliography, without losing track of the main lines of thought at any moment; the result is very pleasant and readable, and constitutes a valuable addition to the literature on the subject.

The second half of the volume makes such a contrast with the first one as to come almost as a shock to an unprepared reader. No better anticlimax could have been imagined to the up to date account of linear algebra given in the first part, than this report on "normal forms of matrices," which, with the few additions needed to include the later literature, might have been lifted bodily from a book written twenty years ago. It seems that here the author chose to follow chiefly MacDuffee's *Ergebnisse* volume, so that he naturally suffers from the defects of his model. Such a book may have had its use in 1933; but, after all the efforts spent in the meantime by the proponents of the conceptual approach to linear algebra on the clarification of the theory, how is one to judge this miscellany of totally unrelated results, brought together for no other reason than their being expressed in terms of matrices? Thus, the notions of eigenvector, eigenvalue, and characteristic determinant are *followed* by results on divisibility of matrices, and this in turn by equivalence and similarity of matrices! Reversing completely the point of view of the first part, all these notions are first given for matrices, and only afterwards linked with intrinsic concepts (the translation of the theorem on elementary divisors of a matrix, in terms of the decomposition of a module in a direct sum of cyclic summands, is given only after more than a page of results expressed in pure matrix language). But if already this type of exposition completely blurs out the essential features of the theory, things become even worse when it comes to the so-called "congruence" of matrices, which occupies the last 10 pages of the book. From the general plan of "Band I" (Algebra and number theory) published by the editors, it appears that these paragraphs are intended as a treatment of the algebraic aspects of quadratic and hermitian forms (a further volume is planned for the "arithmetical" theory of these forms). If such is the case, one can only deplore that the author completely failed to carry out this plan. Quadratic or hermitian forms are not even mentioned, until, after 7 pages, the author, as a kind of remorse, introduces, in an "alternate" proof of a theorem of Toeplitz, the notion of *positive definite* hermitian form! The fundamental relationship between bilinear forms and linear (or semilinear) mappings of a space into its dual is never mentioned at all; nor, of course, any of the fundamental geometric notions of the theory (such as orthogonality, isotropic vectors, Witt's theorem etc.), which maybe are reserved for the sections of the *Enzyklopädie* on "Geometry"? The Clifford algebra (without its name) is given the most cursory treatment in 7 lines; after which we meet a paragraph which begins with the statement that the eigenvalues of a hermitian matrix are real, and in the next sentence deals with Sylvester's law of inertia, as if the two questions belonged to the same theory! One could add many more instances of the absence of any serious attempt to organize the material here presented, but enough has been said to warn the reader that if he looks for clarification of his ideas on the topic, he had better try some other book. Except for their references, this section is, in the opinion of the reviewer, of very little value, and it is to be hoped that the editors of the series will realize their mistake and give the whole subject the treatment it deserves in a later edition.

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