

a given sentence is in this class or not. [This is not yet the same as the word-problem for groups which deals with sentences of a particular form. The word-problem for groups is still unsolved.] The proof is by the indirect method by interpreting a translation of the finitely axiomatized essentially undecidable arithmetic theory in an extension of group theory.

Much credit is also due to the editors of the series on "Studies in logic and the foundations of mathematics" for their part in the publication of this and many other excellent volumes.

ILSE NOVAK GÁL

Pages choisies d'analyse générale. By M. Fréchet. Paris, Gauthier-Villars; Louvain, Nauwelaerts, 1953. 216 pp. 2.000 fr. (\$5.94).

Jubilees or seventieth anniversary celebrations of savants usually carry with them the publication of an issue of a journal dedicated to the celebrant, or a special volume consisting of papers in his main field of interest, contributed by his friends, pupils and admirers. The present volume, which is not in either category, was not planned for in connection with the Jubilee of M. Fréchet. It is, however, an outgrowth of this celebration in that friends and pupils on that occasion urged on him the desirability of issuing a volume which might contain some of the material planned for in a second volume of his *Espaces abstraits*, had not a change of positions and consequent change of fields of interest and research intervened in 1928. The volume under discussion might fall into the category of "excerpts from collected works" which would be insufficiently inclusive in that it is limited to re-publication of some papers connected with the topic of general spaces and not intended to give a systematic insight into the development of these researches. We have then before us a group of papers published in various periodicals, a half dozen of which may be somewhat inaccessible at present. The selection was made by the author and can be considered an indication of what phases of his researches in this field he considers worthy of reproduction and important.

The papers are roughly grouped into chapters headed as follows: 1. Survey of sets (1 paper); 2. Functional spaces (13 papers); 3. Functional analysis (13 papers); 4. Abstract spaces (4 papers); 5. General analysis (6 papers). However, there is no sharp distinction between the topics included in the various chapters. For instance, papers pertaining to the differential are found in the third and fifth chapters, matters pertaining to the generalization of the Weierstrass theorem of approximation in the third and fifth chapters, considerations per-

taining to curves, surfaces, and surface area in the second, third, and fifth chapters. While the reprinted papers are chronologically arranged in each subheading of the chapters, one is aware of much jumping back and forth in point of time, so that papers from various periods rub elbows with each other. There are reprints of items appearing in the very early researches, i.e. 1904, including two excerpts from his famous thesis of 1906, and one paper which appeared in 1950. As a consequence, consecutive reading of the articles in this collection gives one a rather blurred image of the historical development of ideas contributing to general analysis theory on the part of the author.

As is to be expected, the volume contains a number of papers which have been basic and stimulated further mathematical developments. There is the paper of 1915 on the integral of a function over an abstract set generalizing the Lebesgue integral, which incidentally is one place where the points of view of Fréchet and E. H. Moore on general analysis met in the consideration of a functional analysis where the basic class of elements is unconditioned, and one manipulates the real-valued functions on the class. There are two papers on the notion of differential which has proved valuable for instance in implicit function theory on linear normed spaces. There is the paper on bilinear functionals on the space $C \times C$, where C is the space of continuous functions on a linear interval, which introduced an interesting notion of bounded variation for functions of two variables and which has recently been extended and generalized by M. Morse. There are a number of papers on surfaces which have influenced the development of theory of surfaces. There is an excerpt from the 1910 paper on dimension theory proposing the existence of a homomorphism as a basis for an ordering of dimensions.

One is a little puzzled by the insertion of a note headed "malentendu" in the chapter on abstract spaces. It seems to be an apology or defense for having tied the topology of abstract spaces initially to the notion of convergent sequences, i.e. the so-called L (imit) spaces, and later abandoned this type of space for topologies based on vicinity notions. It is not clear why any apology is necessary, since every openminded mathematician knows that initial basic concepts which open up new fields of investigation frequently give way to others which are either better suited to the purpose or receive vociferous support from a prominent member of a mathematical group and his supporters or pupils. Interesting in this same chapter also is the paper "De l'écart numérique à l'écart abstrait," a reaction to the spaces with uniform structure of A. Weil.

There are supplementary remarks after some papers, especially the more important ones, indicating briefly further developments and some of the individuals who have built upon and expanded the material of the original publication.

Although an offset process has been used in the publication of this volume, the papers included have all been retyped, leading to a uniform typography, but introducing occasional unimportant misprints.

T. H. HILDEBRANDT

Trattato di analisi matematica. Vol. 1. By M. Picone and G. Fichera. Rome, Tuminelli. 5700 Lire (\$9.20).

This volume is the first of three which will be devoted to introducing the young student to classical analysis. Into it, Professor Picone, with the very able assistance of Professor Fichera, has poured all the knowledge and experience derived from forty-five years of teaching and some twenty-five as director of Italy's chief computing laboratory. And "poured" is but a meager word to indicate the enormous enthusiasm and energy which have been lavished on the undertaking. The discussion is very detailed and the material elaborate. The book will serve ably not only the budding mathematician but the future physicist and engineer as well. Indeed, being written by men who are outspoken adherents of the "applied school" of mathematics, this treatise is especially recommended to those beginning scientists who frequently complain that most mathematicians ignore their needs.

The present volume treats in its five chapters: matrices and determinants; sets, functions, and continuity; differentiation; integration; and the beginnings of complex analytic functions. The treatment varies from what in the United States would be considered elementary to the level, roughly, of our advanced calculus courses. A principal difference is that the amount of material is more extensive than that which we usually cover. According to the American method, this material is taught twice, once in a general and intuitive fashion to undergraduates, and again in a strictly rigorous fashion for serious or graduate students. The European system (if there is one such) seems to steer an intermediate course of teaching the material once, more thoroughly than we do it the first time and lacking the rigor which we deploy in the second. The superiority of one method over another, we leave for discussion to some more relaxed moment and less public place. National systems of education being what they are, no suggestion is being made that we should adopt this or another similar treatise for teaching our students. However, we would advise the inquiring mind to turn to a book such as this in order to derive some ideas