

$f(x)$ belongs to $M(\phi, p)$ only if $\|f\|_M$ is finite where

$$\|f\|_M = \text{l.u.b.} \left\{ \int_e |f(x)|^p dx / \int_0^{me} \phi(x) dx \right\}^{1/p},$$

e ranging over all measurable subsets of $(0 \leq x \leq 1)$ for which $me > 0$, where me is the measure of e . Many properties are established for these spaces. The chapter concludes with a discussion of Hausdorff summability.

Chapter IV presents the quite difficult theory of the behavior of Bernstein polynomials in the complex domain. The results here are too complicated to admit simple description or illustration.

The exposition of the theory of Bernstein polynomials which this volume contains is quite complete. In collecting together this material, much of it from widely scattered and not easily available sources, the author has performed a valuable service.

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An introduction to abstract harmonic analysis. By L. H. Loomis. New York, van Nostrand, 1953. 10+190 pp. \$5.00.

Abstract harmonic analysis is a branch of mathematics based on the concept of the Fourier-Lebesgue integral transform on the real line, replacing that line by a locally compact topological group G , and the functions e^{ix} by functions arising in representations of G (e.g. characters). The appeal of this rather new and growing doctrine is largely derived from its providing a field of application for concepts and techniques (in point set topology, abstract integration, linear spaces, operators in Hilbert space, and so forth) which had not yet found applications commensurate with their purely aesthetic value.

In any case, this doctrine provides an eagerly desired framework on which may be fastened, for contemplative or expository purposes, many of the more striking ideas of spectral theory and functional analysis. The structural members of this framework are more or less purely algebraic concepts from the theory of groups and their representations, algebras and their ideal theory.

The present work (an outgrowth of courses given at Harvard by G. W. Mackey and later the author), in tacit agreement with this point of view, is built around one of the simplest and yet most valuable topologico-algebraic concepts: the commutative Banach algebra.

The first chapter contains point set theoretic preliminaries, topological products of compact spaces, and Stone's generalization of

Weierstrass' theorem on approximating continuous functions by polynomials.

Banach spaces are considered in the second chapter. The theorem that an operation with closed graph is continuous, as well as the Hahn-Banach theorem even for the complex case, is proved.

The chapter on integration (III) is based on the method devised by Percy John Daniell (1889–1946) in 1917, for elaborating a linear functional ("elementary integral") defined in a vector lattice of functions on a set S . In the definition of the L^p spaces, technical convenience is attained by admitting only Baire functions. Finally, the case of locally compact S is considered, and integration over product spaces (Fubini's theorem) is established for that case.

The introduction to Banach algebras (Chap. IV) requires close attention of the reader. Although the Banach algebras really needed later are commutative (except for the H^* -algebras), the discussion leaves the way open to wider applications. Thorough use is made particularly of the device of considering algebras of complex functions on arbitrary sets, generalizing the algebra of functions on the maximal-ideal space of the commutative case. The broad approach provides occasions for making variant definitions of essentially the same concept; in some cases the variations are apparently made tacitly. For example the reader must guess what Δ on p. 55 is, and amplify the propositions there.

Banach algebras without units are treated in Segal's manner, with regular ideals, adverses, etc. The adjunction of a unit element is considered but regarded as impractical. (In any case, the theorem about the enlarged ring on p. 59 is inaccurate, and should be limited to maximal ideals or commutative algebras.) Segal's treatment of the kernel (of a set of maximal ideals) and the hull (-set of an ideal) is included. The uniqueness of norm, spectral radius, application of analytic mappings within the algebra, are also presented in this chapter.

The succeeding one takes up commutative semi-simple Banach algebras which are *regular* (where this word now means that the sets $\{\delta; \delta(x) \neq 0\}$, x in A , form a basis for the space Δ of continuous homomorphisms of A on the complex numbers). It is characteristic of the arrangement of the book that a Tauberian-type theorem (about frontiers of hulls) is next proved rather than after the introduction of the L^1 algebra (which is the main application). Of course, suitable forward-looking references are always made at such points. The full functional representation theorem for commutative C^* -algebras and the spectral theorem come next. Then for *self-adjoint* algebras (e.g., if there is a suitable involution) there is proved a Herglotz-

Bochner-Weil-Raikov theorem on positive functionals, and a Plancherel theorem (after Godement). Thus the later proofs of the more familiar form of these theorems for group algebras become spectacularly abrupt.

Haar measure (with a neat treatment of quotient measures) and the Banach algebra of L^1 under convolution over a locally compact group having been disposed of in Chapter VI, the next chapter gives that satisfactory treatment of the theory of characters for the abelian case which the concept of maximal ideals makes possible. Pontrjagin's duality theorem itself is rather ignominiously embedded in a section on miscellaneous theorems, and provides an occasion for remarking that the important structural considerations on which this duality theorem was based in pre-normed ring days are not (as contrasted with A. Weil's book) here presented.

Compact groups and almost periodic functions are next treated by means of Ambrose's H^* -algebras, which were (of course) set up in an earlier chapter. The work closes with a stimulating chapter on further developments. A paper of H. J. Reiter, whose place of publication was indefinite at the time, is in the *Trans. Amer. Math. Soc.* vol. 73 (1952) pp. 401-427 and further references of interest are in R. Godement's paper, pp. 496-556 in the same volume.

The author is certainly to be thanked for his efforts in producing, in this field, a text which will take its place next to Pontrjagin's and A. Weil's in the literature of locally compact groups. Aesthetic considerations certainly played a large part in shaping the exposition. It is to be hoped that the book will be widely used as a text for graduate courses. I would guess that there was material for about 75 lectures of the usual sort. The instructor will have to consider each lecture carefully in advance, think about adding to the index (an index of symbols would help), and provide alternative motivations with detailed examples. The student will understand that this monograph is not a treatise on the fields partly covered by the chapters on prerequisites (Banach algebras, operator algebras, topological groups, etc.). Nevertheless, after careful study of this book, he should be able to decide whether he shares the ability of some, and the enthusiasm of many, for research in these fields.

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Conformal representation. By C. Carathéodory. 2d ed. Cambridge University Press, 1952. 10+115 pp. \$2.50.

Except for the addition of a chapter on the uniformization theorem, this is an exact reprint of the original 1931 edition (which, by an oversight, was never reviewed in this Bulletin).