

will define the additional coefficients more precisely and this were the only outcome of the book, it would indeed have served its purpose.

L. HOWARTH

A first course in functions of a complex variable. By W. Kaplan. Cambridge, Mass., Addison-Wesley, 1953. 8+485-620 pp. \$3.50.

The publishing of this text represents a new step by the publishing industry, taken presumably to help combat the sharply increasing price of textbooks. The author's *Advanced calculus* (A) published in 1952 contained a long chapter on complex variables, and it is this chapter now in separate binding that is offered at \$5.00 less than the price of A. The only alteration, aside from the correction of misprints, is the addition of an appropriate index, a subset of the index of A.

The author has an attractive style of writing, the material is well organized and in a natural order, and despite the limited space available the author covers an amazing amount of material, including even a brief account of the applications of conformal mapping to the theory of elasticity. Answers are provided for all numerical exercises, a remarkable and refreshing innovation for advanced textbooks. On the debit side the treatment of the Riemann sphere and the linear fractional transformation is too brief, and the cross ratio is relegated to the exercise list.

Although the book has considerable merit and would certainly be a satisfactory text for a first course, the central question to be considered is the publisher's decision to reprint one chapter of A. Despite the author's assertion that "Outside of a few references to earlier chapters of A this book is essentially self-contained," there are a rather large number of places where the student who does not possess a copy of A will be at a distinct disadvantage. To cite only one such case, the author gives the Cauchy-Hadamard formula for the radius of convergence of a power series, and then for a proof states that "Since the ratio test, root test, and M -test all carry over to complex variables . . . , the proof given in Section 6-15 can be repeated without change." But Section 6-15 is not in the complex variable chapter.

Although recent years have seen a substantial increase in the number of texts on complex variables available, there is still a need for a good elementary one. The author has here an excellent start, but the reviewer hopes he will continue the present book in both directions and expand it in the middle so that on the one hand it will really be

self-contained, while on the other hand the student who wishes to pursue the subject further can do so.

A. W. GOODMAN

Bernstein polynomials. By G. G. Lorentz. (Mathematical Expositions, no. 8.) University of Toronto Press, 1953. 10+130 pp. \$5.75.

For a function $f(x)$ defined for $0 \leq x \leq 1$ the formula

$$B_n(x) = \sum_{\nu=0}^n f\left(\frac{\nu}{n}\right) \binom{n}{\nu} x^\nu (1-x)^{n-\nu}$$

defines the Bernstein polynomial of order n of $f(x)$. Evidently $B_n(x)$ is a polynomial in x of order less than or equal to n . S. Bernstein, who introduced these polynomials, proved that if $f(x)$ is continuous for $0 \leq x \leq 1$, then

$$\lim_{n \rightarrow \infty} B_n(x) = f(x)$$

uniformly for $0 \leq x \leq 1$, thus obtaining a particularly simple and elegant proof of the Weierstrass approximation theorem. Bernstein polynomials are of interest in themselves, and in addition play an important role in several other mathematical topics, notably in the finite moment problem and in the related theory of Hausdorff summability.

Chapter I deals with the approximation of continuous functions by Bernstein polynomials. A great many results are proved of which the following may serve as an example: if $f(x)$ satisfies a Lipschitz condition of order α , $0 < \alpha < 1$, then

$$|f(x) - B_n(x)| = O(n^{-\alpha/2}) \quad (n \rightarrow \infty),$$

uniformly for $0 \leq x \leq 1$. Care is taken to explain the relation of these results to the general theory of polynomial approximation.

Chapter II treats the changes which are necessary if $f(x)$ is no longer continuous, but merely integrable, or even only measurable. For example if $f(x)$ is integrable $B_n(x)$ may be replaced by

$$P_n(x) = \sum_{\nu=0}^n \binom{n}{\nu} x^\nu (1-x)^{n-\nu} \left[(n+1) \int_{\nu/(n+1)}^{(\nu+1)/(n+1)} f(t) dt \right].$$

It is shown that

$$\lim_{n \rightarrow \infty} P_n(x) = f(x)$$