stages of polishing and refinement until it becomes the bloodless, dead form of "Definition, theorem, proof" in manifold repetition. Originally to be found in Euclid's *Elements*, this striving for formalism has been revived in modern times—"chiefly in North America," but, partly as a consequence of Hilbert's work and partly because of the impact of American tastes, also influencing mathematical work in Europe. Whether one agrees with this or not, one cannot but feel that the author has a point in regard to that excessive formalizing and abbreviation whose only justification is to comply with demands for brevity due to high printing costs.

By mathematics as "*power*," the author is careful to make clear that he means not material power or power over one's fellow men, but the feeling that comes, for instance, with the creation of a mathematical tool that enables one to solve a whole class of problems as special cases of a general theory.

Although there is plenty of room for disagreement with some of the author's principal assertions (as well as with some minor ones such as the mention of Leibniz as, by implication, the sole creator of the calculus), this little book is to be highly recommended as general reading for the professional mathematician and as "must" reading for the layman whose notions of mathematics are embodied in the query, "Won't you please add up my bridge score; you're a mathematician, aren't you?"

R. L. WILDER

An introduction to homotopy theory. By P. J. Hilton. Cambridge University Press, 1953. 8+142 pp. $3.00.

This is the first book to appear on homotopy theory. There has long been a need for collecting results from diverse sources, and this book seems to meet a large part of that need. Of course, a book of this size cannot be comprehensive and other books will be required. It is surprising, however, how much the author manages to get into the small number of pages. This seems to have been accomplished by careful planning and abetted by the author's ability to explain what is happening without becoming verbose. Another factor is the restriction to homology theory of complexes with integer coefficients, although one might argue that the introduction of singular theory would pay for itself in simplified proofs (and more general theorems) in some cases.

The book seems well suited for a textbook even though it does not contain exercises. The most important factor is that it can be read by a student. Then there is what should turn out to be a good pedagogic organization of the material. For example, some information on the homotopy groups of the rotation groups is obtained from basic properties of fibre spaces. Later the problem is resumed after the Hopf invariant and Freudenthal suspension have been introduced. Similarly, a special case of the Whitehead product is introduced and used in a proof considerably before its general definition is given. In both cases the reader is told exactly what is happening. Most of the theorems are proved. The exceptions are usually easily available.

The first chapter is a short introduction which defines the concept of a homotopy between two maps (arcwise connected Hausdorff spaces are used throughout) and the concept of homotopy type. This latter is proved to be an equivalence relation between spaces.

Chapter II defines the homotopy groups. First the absolute groups are defined using cubes, then the alternative description in terms of maps of spheres is shown to be equivalent. The operation of π_1 on π_n is given and n-simplicity of a space is defined. It is proved that the homotopy groups are invariants of homotopy type. Next the relative groups are defined both ways and equivalence is established. These are shown to be invariants of the relative homotopy type. In defining the operation of π_1 on π_n the "top hat" lemma was used, and it is now extended to the homotopy extension theorem.

The next chapter gives the basic theorem on the Brouwer degree of a map of S^n to S^n and the Hurewicz isomorphism theorem (relative). Since singular theory is not used, it is necessary to have the simplicial approximation theorem, and the Hurewicz isomorphism theorem is stated only for complexes. For proofs the reader is referred to Lefschetz's *Introduction to topology*.

In Chapter IV the homotopy sequence of a pair is defined and proved to be exact. A map between pairs is shown to induce an operator homomorphism of the sequences (with π_1 of subspaces as operators). If a pair consists of complex and subcomplex a homomorphism is obtained from its homotopy sequence to its homology sequence. The direct sum theorems obtainable from the homotopy sequence in special cases (e.g. the subspace is a retract of the space) are given. The homotopy sequence of a triple $C\subset B\subset A$ is defined and a sample of the exactness proof is given. There is a section on $\pi_2(Y, Y_0)$. This group is not necessarily abelian, but it is shown to be a crossed module under the operation of $\pi_1(Y_0)$.

Chapter V is on fibre-spaces. These are defined by local triviality (condition (ii) is the sole condition since it implies (i)), and no structure group is used. This condition suffices to give the strong "covering extension theorem." Let $\phi: X\rightarrow Y$ be such a fibre-space and

L be a subcomplex of the finite complex K and such that L is a deformation retract of K. If $f_0:L\to X$ is such that $g_0=\phi f_0:L\to Y$ admits an extension to $g:K\to Y$, then f_0 admits an extension $f:K\to X$ such that $g=\phi f$. This theorem implies the covering homotopy theorem and gives the exact homotopy sequence of the fibre-space. The exact sequence is used to study fibre-spaces over spheres. The last section considers the definition of fibre-spaces (here called pseudo-fibre spaces) by using the covering homotopy theorem as the sole axiom. For these one gets the covering extension theorem only in case K is contractible. It is shown that the space of paths between two subsets A, B of Y is such a fibre-space over $A\times B$. The technique of representing $\pi_n(Y)$ as the one-dimensional homology group of a certain space of loops is mentioned. The Cartan-Serre construction of the Eilenberg-MacLane spaces $K(\pi, n)$ is given.

The sixth chapter is entitled "The Hopf invariant and suspension theorems." The Hopf invariant γ of a map $S^{2n-1}\to S^n$ is defined and shown to depend only on the homotopy class of the map. If n is odd $\gamma=0$, and if n is even $\pi_{2n-1}(S^n)$ has an element with $\gamma=2$. The Freudenthal suspension and the J. H. C. Whitehead generalization are defined and the suspension theorems are stated. These are applied to fiberings by spheres and Stiefel manifolds. Finally the author's generalization of the Hopf invariant is given as $H^*=\chi Q\mu:\pi_r(S^n)\to\pi_{r+1}(S^{2n})$ all r, n. Here $\mu:\pi_r(S^n)\to\pi_r(S^n\cup S^n)$ is induced by shrinking the equator, $Q:\pi_r(S^n\cup S^n)\to\pi_{r+1}(S^n\times S^n, S^n\cup S^n)$ is projection onto a direct summand, and $\chi:\pi_{r+1}(S^n\times S^n, S^n\cup S^n)\to\pi_{r+1}(S^{2n})$ is induced by shrinking $S^n\cup S^n$ to a point. This is shown to be a true generalization and its relation with the G. W. Whitehead generalization is given.

Chapter VII gives a discussion of CW-complexes. It includes the Massey homology spectrum and takes from this spectrum the exact sequence which J. H. C. Whitehead used in classifying simply connected 4-polytopes according to homotopy type. For most of the results the reader is referred to *Combinatorial homotopy.* I (J. H. C. Whitehead, Bull. Amer. Math. Soc., 1948). However, a proof is given of the important theorem stating that if $f:K\to L$ realizes an isomorphism between the homotopy groups of the CW-complexes K and L, then f is a homotopy equivalence.

Chapter VIII considers the problem: If K is a CW-complex such that $\pi_i(K)=0$ for $i=0, 1, \cdots, n-1$ $(n>2)$, what is $\pi_{n+1}(K)$? This involves a detailed study of a section of the exact sequence obtained in Chapter VII. When $\dim K\leqq n+2$, $\pi_{n+1}(K)$ is obtained as a definite group extension of $H_{n+1}(K)$ by a certain subgroup of $\Gamma_{n+1}(K)$

$=i_*(\pi_{n+1}(K^n))$ where $i:K^n \rightarrow K^{n+1}$ is the identity map. When, in addition, $\pi_{n+1}(K)=0$ and $n>3$, $\Gamma_{n+2}(K)$ is computed to be $H_n(K)/2H_n(K)$. This is equivalent to the statement that $\pi_{n+2}(K) \approx H_{n+2}(K)+H_n(K)/2H_n(K)$, and in this form it is proved again using the normal cell complexes of S. C. Chang.

There is a bibliography of 4 books and 42 papers and a rather complete combined index and glossary.

M. L. CURTIS

Lezioni sulle funzioni ipergeometriche confluenti. By F. G. Tricomi. Torino, Gheroni, 1952. 284 pp. 2000 Lire.

Die konfluente hypergeometrische Funktion mit besonderer Berücksichtigung ihrer Anwendungen. By Herbert Buchholz. (Ergebnisse der angewandten Mathematik, vol. 2.) Berlin-Göttingen-Heidelberg, Springer, 1953. 16+234 pp. 36.00 DM.

Confluent hypergeometric series arise when two of the three singularities of the hypergeometric differential equation coalesce in such a manner as to produce an irregular singularity at infinity. These series were introduced by Kummer in 1836. In 1904, E. T. Whittaker proposed new definitions and notations which clearly exhibit the symmetry and transformation properties of confluent hypergeometric functions, and facilitate the identification of many special functions, among them Bessel functions, Laguerre and Hermite polynomials, error functions, Fresnel integrals, sine and cosine integrals, exponential integrals, and the like, as particular instances of confluent hypergeometric functions. (As a matter of fact, the chapter on Bessel functions in all newer editions of Whittaker and Watson's *Modern analysis* follows upon, and leans heavily on, the chapter on confluent hypergeometric functions.)

In the first half of the present century a sizeable literature has grown up around these functions. Many of their properties were discovered (some of them several times), and they have been found useful in pure and applied mathematics alike. Fluid dynamics, nuclear physics, probability theory, elasticity, all offer problems which can be solved in terms of confluent hypergeometric functions, and these functions proved an excellent example for illustrating the technique of the Laplace transformation. In view of the hundreds of papers and dozens of applications it is somewhat strange that no monograph on these functions seemed to be available (although several well-known books, such as *Modern analysis*, Jeffreys and Jeffreys' *Methods of mathematical physics*, and Magnus and Oberhettinger's *Special functions* devote separate chapters to these functions). Now, within a