

A university algebra. By D. E. Littlewood. London, Heineman, 1950. 8+292 pp. 25s.

Something less than the first half of this book is concerned with the same kind of subject matter as in the text by Stoll reviewed above. However, Littlewood's matrix theory goes beyond Stoll's (and beyond most standard treatments) by including a great deal of material on the theory of determinants, which in turn motivates a smattering of permutation group theory and a large amount of detail on symmetric and alternating polynomials (all this presumably with an eye to applications to group representations later in the book). The next third of the book contains a more or less standard account of the theory of equations, some elementary theory of numbers, a little field theory in connection with the subject of polynomials, and a short chapter on the Galois theory of equations (by resolvents). The remainder of the book is devoted to the more advanced topics of invariants and group representations, the latter attacked via the Wedderburn theory of algebras, ending with a discussion of the irreducible characters and representations of the symmetric, general linear, and orthogonal groups.

The text is pitched at the advanced undergraduate or beginning graduate level. The calculational sections and the examples are worked out in detail, and the book is well supplied with exercises. On the other hand, the theoretical sections of the text tend to be condensed or slurred over, making them quite obscure at times, and of dubious accessibility to the uninitiated.

The fundamental approach in this volume is almost the opposite of Stoll's. In matrix theory, for example, the center of the stage is occupied by n -tuples and matrices, with their attendant canonical bases, rather than by abstract vector spaces and linear mappings thereon. This lack of what one might call a geometrical approach results, of course, in a computational matrix theory, in contrast with the more conceptual linear transformation theory where no basis is a preferred one. However, algebra and geometry, which had been divorced by decree in the very first section, are reconciled at last in one of the late chapters, on the theory of invariants, which theory "eliminates all the heavy and irrelevant work which the superfluous frame of reference carries with it, and restores the simplicity and elegance which are characteristics of pure geometry."

Like Stoll, Littlewood introduces the concepts of group, ring, integral domain, and field in short parenthetical sections when the concepts arise naturally. However, this leaning toward "abstract algebra" is perfunctory. For example no definitions are given for

“vector space” (although “algebra” is defined) or “quotient group” (although the phrase and the concept are ultimately used, without warning); definitions of abstract concepts that are given are almost never linked to one another (e.g., the underlying additive group of a ring or field is not mentioned); in fact, the only groups that are treated as such are groups of permutations or matrices.

Lastly, we should mention a third contrast between the present text and the one above. A fatal carelessness pervades the whole book, even when abstract concepts are only in the background. To give two examples, the reader is hard put to it to discover in any particular section whether “number” means real number or complex number (see especially the section on orthogonal matrices); linear dependence of a set of vectors is defined incorrectly (the phrase “scalars not all zero” is omitted) and so is linear dependence of one vector on a given set of vectors (the phrase “scalars not all zero” is included). This kind of carelessness finally leads the author into at least two real errors: (1) A gap in the proof of Wedderburn’s theorems, where the author uses without proof the existence of a unit element in a semi-simple algebra, the “justification” presumably being this blanket statement a few sections back: “If an algebra does not possess a modulus [unit element] then a modulus may be *adjoined*. . . . It will be assumed henceforth that if necessary a modulus has been adjoined.” (2) Theorem II, Chapter X, p. 156 is false. It reads: “If $F(\xi_1, \xi_2)$ is the extension field obtained by adjoining to F two roots ξ_1, ξ_2 of an irreducible equation in F , then the correspondence $\xi_1 \rightarrow \xi_2, \xi_2 \rightarrow \xi_1$ constitutes an automorphism of the extension field [over F].” A counter-example is provided by any cyclic extension field of degree > 2 .

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Linear transformations in n -dimensional vector space. An introduction to the theory of Hilbert space. By H. L. Hamburger and M. E. Grimshaw. Cambridge University Press, 1951. 10+195 pp. \$4.50.

Since the purpose of this book is clearly indicated by its title and subtitle, this review can consider first the order (quite reasonable) and clarity (good) with which the authors present their material.

The vector space V_n considered is always the space of sequences of n complex numbers. The scalar product, linear dependence, linear subspaces, orthogonal complements, and the algebra of linear transformations are discussed in Chapter I. Chapter II describes the elementary properties of Hermitian, normal, unitary, and projection operators and their eigenmanifolds. Chapter III derives the spectral