was not referred to in the preceding section; but "these results are true regardless of whether the domains in question are simply-connected or multiply-connected." He does at least point out here that one does have to worry about single-valued conjugate harmonic functions in multiply-connected domains. Another example appears when the Cauchy integral theorem is proved for functions with a derivative on the boundary and later, in proving the symmetry principle, is used for functions only continuous on the boundary. Also in Chapter VII, the author claims "to prove the existence of the solution of the Dirichlet problem in the case of a general domain of finite connectivity," not stating how *general*. In reality, he proves it for domains bounded by smooth Jordan curves. The reviewer regrets that the lucid style of the author was not utilized to give a textbook presentation of the theory of prime ends, which would have been appropriate in a book of this type and is lacking in the literature. It is also rather unfortunate that the author followed the only too prevalent custom of claiming that this work, being a textbook, need not be documented. No references to original sources or to other works are given. This is true even when, in general discussions, the author makes statements he does not prove in the book. The value of this excellent book to the graduate student would have been enhanced considerably if it had also furnished a key to further study.

G. SPRINGER

The theory of functions of a real variable. By R. L. Jeffery. (Mathematical Expositions, No. 6.) University of Toronto Press, 1951. 13+232 pp. $6.00.

This book consists of two distinct parts. The first part (Chapters I-V) gives a general introduction to functions of a real variable, measure, and integration, while the second part (Chapters VI and VII, with Chapter VIII as a kind of appendix) treats the problem of inverting the derivative of continuous functions, leading to the Denjoy integrals, and studies the derivates and approximate derivates of functions of a real variable on arbitrary linear sets. The author himself, who in previous papers has made some valuable contributions to these topics, considers the presentation of this second part as the main purpose of his book. In both parts only functions of one real variable are discussed.

After an introduction concerning the real number system, Chapter I deals with sets, sequences, and functions and Chapter II with metric properties of sets. Here the author considers only the outer measure of a set A, called by him the metric of A and designated by $|A|^0$. Two sets A and B are metrically separated if for $\epsilon>0$ there exist two open sets $\alpha\supset A$ and $\beta\supset B$ with $|\alpha\beta|^0<\epsilon$. If A and its complement are metrically separated, then A is measurable and $|A|^0$ is designated by $|A|$, the measure of A. In the statement of Theorem 2.15 one has to add the condition "provided that $|A_n|$ is finite for some n." Chapter III introduces the Lebesgue integral in the classical Lebesgue manner. Immediately after the definition of the Lebesgue integral, "for purposes of comparison and contrast," the Riemann integral is also defined and discussed in one short section. Later, an analogous representation of the Lebesgue integral is given, depending upon a subdivision of the domain on which the function is defined. Chapter IV discusses properties of the Lebesgue integral, including integrability of sequences and integrals containing a parameter, and finally gives a detailed proof of the ergodic theorem. In Chapter V the Vitali covering theorem is first proved and then applied to a discussion of metric density of sets and to the proof of the theorem that a functon $f(x)$ of bounded variation on a set A is differentiable almost everywhere on A; the summability of this derivative is also proved. This chapter is concluded by a short introduction to functions of sets.

Chapter VI gives a systematic and thorough discussion of the problem of inversion of derivatives. In particular, the case of a finite, but non-summable, derivative of a continuous function $F(x)$ in an interval $[a, b]$ is studied; $F(x)-F(a)$ can here be determined by at most denumerably many operations on $F'(x)$, and so the general Denjoy integral, in the very comprehensive form of W. H. Young, is obtained. Finally, descriptive definitions of the special and general Denjoy integrals are discussed; in this last section of Chapter VI some results are only stated, but not proved.

Chapter VII is devoted to the study of derivates and approximate derivates (or, as the author says, approximate derived numbers). Besides the Weierstrass nondifferentiable function, the Besicovitch function without unilateral derivatives is also discussed, in the interpretation given to it by E. D. Pepper, but with different methods. Then, for arbitrary functions $f(x)$ defined on arbitrary sets, the four principal derivates and their mutual relations, as well as the approximate derivates and their relations, are studied systematically. The author here essentially gives a detailed discussion of the very general results obtained by him in Ann. of Math. vol. 36 (1935) pp. 438–447. He now modifies his definition of the four principal approximate derivates of $f(x)$ on a set A, bringing it into the following more concise and inclusive form: The upper right approximate derivate of $f(x)$ at the point x over the set A, D^+f, is the supremum of the numbers a for which

$$\frac{f(\xi) - f(x)}{\xi - x} \geqq a, \qquad \xi > x, \qquad \xi \in A,$$

for a set ξ of right density greater than zero at x. For the discussion of these approximate derivates the notion (already used by him, loc. cit.) of a function $f(x)$ metrically separable relative to the set A is essential; that is, for every a the two sets $A\,[f(x) < a]$ and $A\,[f(x) \geqq a]$ have to be metrically separated. At the end of this chapter, relations to H. Blumberg's investigations on arbitrary functions [Acta Math. vol. 65 (1935) pp. 263–282] are also outlined.

The last Chapter VIII discusses the Riemann-Stieltjes integral and, in connection with it, linear functionals. The author finally indicates how this leads to the idea of the distributions of L. Schwartz.

So we see that many interesting topics are treated in this book. Moreover, the presentation is very careful and readable.

ARTHUR ROSENTHAL

BRIEF MENTION

Einführung in die Funktionentheorie. By L. Bieberbach. 2d ed. Bielefeld, Verlag für Wissenschaft und Fachbuch, 1952. 220 pp., 43 figs. 12.60 DM.

The present volume is a second revised edition of Bieberbach's well known *Einführung in die Funktionentheorie.* It is intended for the reader who possesses a modicum of classical real analysis. Applications of complex variable methods to hydrodynamics and potential theory are treated. In general, emphasis is put on those aspects of classical complex function theory which are of interest in the applications. In addition to the usual standard material of a first course on the theory of functions of a complex variable, there is a section on practical aspects of conformal mapping which includes a brief account of Bergman's orthogonalization methods.

A number of sections are followed by supplementary material which serves in part as exercises and in part as indications of further developments of the subject matter treated in the corresponding section. In this connection the proof due to Ankeny of the fundamental theorem of algebra which is based directly on the Cauchy integral theorem is worth mention (pp. 85–86).

MAURICE HEINS