These very detailed tables are likely to become an indispensable tool for the practical application of the double Laplace transformation.

The publishers must be congratulated on the excellent performance of a typographical job (the tables) which presents considerable technical and financial difficulties.

A. ERDÉLYI

Mathematische Grundlagenforschung. By Arnold Schmidt. (Enzyklopädie der Mathematischen Wissenschaften, Band I_1, Heft 1, Teil II.) Leipzig, 1950. 48 pp.

This article was originally written in 1939, but was revised in 1948–49, so as to include reference to more recent contributions. It consists of an exposition of "those parts of foundation studies which are either directly concerned with the construction of mathematics, or in which the application of mathematical methods has proved fruitful." The author confines himself almost entirely to the foundations of arithmetic, and does not attempt to deal with such topics as set theory, group theory, or geometry, nor with the problems proper to mathematical logic itself.

The article consists of four parts: (A) *Axiomatik und allgemeine Beweistheorie*; (B) *Kodifikation und Beweistheorie der Zahlenlehr*; (C) *Die logische Begrundung der Mathematik*; and (D) *Intuitionistische Mathematik.*

In Part (A), the author explains what is meant by the formalization of a mathematical system, and introduces some metamathematical terms. He then shows (following Gödel) that every system which contains arithmetic also contains its own syntax in arithmetical form, and proceeds to sketch a proof of Gödel's theorem (as strengthened by Rosser): that a system which contains its own syntax in arithmetical form cannot be both consistent and complete. Among other results of a negative character mentioned here are a second theorem of Gödel, that a consistent system containing its own syntax in arithmetical form cannot be shown to be consistent by any proof which can be formalized within the system, and the theorem of Tarski that, in a system which contains its own syntax in arithmetical form, one cannot define truth for the system itself.

Part (B) begins with a brief sketch of the theory of recursive functions. The result of Péter is cited, that one can keep on getting new recursive functions by increasing the number of variables in primitive recursions. Some of the various equivalent methods of defining general recursive functions are indicated: the original method of Herbrand and Gödel, Turing's notion of computability, and Church's notion of λ-definability. The author mentions Bernays' proof that every function whose values can be calculated in some consistent deductive calculus is general recursive, and describes the normal form of Kleene for general recursive functions. The relationship between general recursive functions and decision methods is pointed out, and reference is made to Church's proof that there is no decision method for the restricted predicate calculus; the more recent extensions of this result by Mostowski, Post, Julia Robinson, R. M. Robinson, and Tarski are not mentioned, however, as they have appeared since Schmidt's article.

The remainder of this part is taken up with an exposition of consistency proofs of various fragments of arithmetic, Gentzen's consistency proof of all of arithmetic using transfinite induction, and a sketch of some unpublished transfinite proofs (due to Lorenzen and Schütte) of the consistency of part of analysis.

Part (C), which is very short, is devoted to a résumé of the theory of types, and of the attempt of Russell and Whitehead (following Frege) to construct arithmetic from logic alone. The author criticizes this attempt on the ground that Russell and Whitehead were unable to prove certain theorems of arithmetic without using axioms which hardly seem to be part of "pure logic" (the axiom of infinity and the axiom of choice).

Part (D) is devoted to an exposition of the intuitionistic mathematics of Brouwer and his followers. The philosophical background of this movement is made clearer here than in the usual presentations.

J. C. C. McKinsey

Dirichlet's principle, conformal mapping, and minimal surfaces. By R. Courant. With an appendix by M. Schiffer. (Pure and Applied Mathematics, vol. 3.) New York, Interscience, 1950. 14+330 pp. $4.50.

At the beginning of its history, Dirichlet's principle was one of the occasions for the ascetic enterprise of revising and clarifying the foundations of the calculus of variations. In its latest stage it has proved a powerful tool for the solution of one of the most interesting, difficult, and colorful problems, a solution which could be carried to a generality which, a quarter of a century ago, even the most fanciful optimism would hardly have dared to dream of. No other mathematician is more competent to write a presentation of this subject than is the author of the present book: His first steps in research