tails to the appendices, which makes the body of the book accessible to a wider class of readers, has inevitably made a smooth and coherent presentation almost out of the question. (2) The discussion of the various applications are quite repetitive, and while this may be desirable for the general reader, it does not help the mathematician. (3) The book is now two years old, and was written when many parts of the field were undergoing very active development. It is to be hoped that future editions, or a book at a higher level, will provide a more coherent account, covering such new work as that of Wolfowitz [5] and Seth [6] on sequential estimation, Wald [3, 4] and Arrow, Blackwell, and Girshick [7], on multiple decision functions.

The book is relatively free from typographical errors. We may mention that a "Δ" should appear in the denominator of the fraction appearing at the very end of page 9, that in expression (3.3) on page 38 the inequality should be reversed, that in expression (4.22) on page 85 the fraction on the right should be inverted, and that the summand of the sum in the third line of page 133 should be squared.

REFERENCES

1. A. Wald and J. Wolfowitz, *Optimum character of the sequential probability ratio test*, Ann. Math. Statist. vol. 19 (1948) pp. 326–439.

2. Statistical Research Group, Columbia University, *Sampling inspection*, New York, McGraw-Hill, 1948.

3. A. Wald, *Foundations of a general theory of sequential decision functions*, Econometrica vol. 15 (1947) pp. 279–313.

4. ———, *Statistical decision functions*, Ann. Math. Statist. vol. 20 (1949) pp. 165–205.

5. J. Wolfowitz, *The efficiency of sequential estimates and Wald's equation for sequential processes*, Ann. Math. Statist. vol. 18 (1947) pp. 215–230.

6. G. R. Seth, *On the variance of estimates*, Ann. Math. Statist. vol. 20 (1949) pp. 1–27.

7. K. J. Arrow, D. Blackwell, and M. A. Girshick, *Bayes and minimax solutions of sequential decision problems*, Econometrica vol. 17 (1949) pp. 213–244.

MELVIN P. PEISAKOFF AND JOHN W. TUKEY

Variétés abéliennes et courbes algébriques. By A. Weil. Paris, Hermann, 1948. 165 pp.

This is the second of a series of papers with which the author promised to follow his book, *Foundations of algebraic geometry*, American Mathematical Society, 1946. The first paper, entitled: *Sur les courbes algébriques et les variétés qui s'en déduisent*, is concerned in particular with the theory of correspondences of an algebraic curve and contains the author's proof of the Riemann hypothesis over a finite field. In a sense this study is generalized to the case of general abelian varieties in the present work.

In the classical sense an abelian variety is a variety in a complex projective space whose nonhomogeneous coordinates are $2n$-ply periodic meromorphic functions in n complex variables. An essential feature is the possibility of defining a group operation on the variety, which the author takes as point of departure in his theory of abelian varieties over a field of characteristic p. Varieties are abstract varieties in the sense of the author, and a function in a variety U, with values in another V, is defined by a sub-variety Z of $U\times V$ such that $[Z:U]=1$. The values of the function are defined only at points where the projection of Z into U is regular. With the notion of function so defined, a group variety G is one such that a function is defined everywhere in $G\times G$, with values in G, with the usual group axioms satisfied. If the variety is complete, then the group operation has to be commutative, a result analogous to compact complex Lie groups. An Abelian variety is a complete group variety.

The classical theory of abelian varieties, in the hands of Scorza, Rosati, Lefschetz, and Albert, is essentially a theory of Riemann matrices, the period matrices of the periodic functions, and one of the main problems is to study the complex multiplication of Riemann matrices. To arrive at these concepts in the abstract approach we consider functions f defined in an abelian variety B, with values in an abelian variety A. Up to an additive constant f preserves group addition. Such a function is called a homomorphism of B into A, and an endomorphism if $B=A$. With addition defined by adding the function values and multiplication defined as successive application of the endomorphisms all endomorphisms of A form a ring $\mathcal{A}(A)$, the ring of endomorphisms. This ring $\mathcal{A}(A)$ can be imbedded into a ring $\mathcal{A}_0(A)$ over the rational field Q. $\mathcal{A}_0(A)$ is a semi-simple algebra over the rational field. If A is a simple variety, that is, one which contains no proper abelian sub-variety, $\mathcal{A}_0(A)$ is a field of finite rank over Q. The study of the rings $\mathcal{A}(A)$ and $\mathcal{A}_0(A)$ takes the place of complex multiplication.

The construction of abelian varieties of a given dimension is achieved by the consideration of the Jacobian variety of a curve. Let Γ be a complete curve without multiple point, of genus $g>0$. It is possible to construct an abelian variety J^g and a function ϕ, defined on Γ, with values in J, such that if $M_1, \cdots, M_g$ are g independent generic points of Γ (with respect to a field of definition K for Γ, J, and ϕ), the point $z=\sum_{i=1}^{g}\phi(M_i)$ is generic on J with respect to K. The Jacobian varieties furnish concrete examples of abelian varieties of a given dimension and give a link between abelian varieties and curves.

But the more important link is furnished by the application of abelian varieties to the theory of correspondence of curves. In fact, there exists an isomorphism between the module of classes of correspondences between Γ and Γ' and the module of homomorphisms of J into J'. In the case $\Gamma=\Gamma'$ this isomorphism is one between the ring of classes of correspondences on Γ and the ring of endomorphisms of J. Thus the study of endomorphisms of an abelian variety is a generalization of the theory of correspondences of a curve.

The above gives perhaps a broad outline of some results of this book in relation to the classical theory. The author has not only generalized the classical theory into a more profound new theory, with new results, but has presented the results in such a way that the development seems most natural. Among other things the book gives ample justification of the struggle one has to go through in reading the *Foundations* of the author.

SHIING-SHEN CHERN

Lattice theory. By G. Birkhoff. (American Mathematical Society Colloquium Publications, vol. 25.) Rev. ed. New York, American Mathematical Society, 1949. 14+280 pp. $6.00.

In the preface to the first edition of *Lattice theory*, Professor Birkhoff remarked that one of the attractive features in writing such a book was "fitting into a single pattern ideas developed independently by mathematicians with diverse interests." Thus the first edition contained a quite exhaustive account of those topics in mathematics which make extensive use of lattice operations. The same philosophy prevails in the new edition though it is a complete revision of the original. Due to the large number of contributions to the subject in the intervening years, the new volume is nearly twice the size of the old, and yet many important topics are barely mentioned.

The general plan of the book is unchanged. Beginning with partially ordered sets (Birkhoff now uses the term "partly ordered set," though he was not completely successful in changing every "partially" into "partly"), the author treats successively more special systems concluding with chapters on lattice-ordered groups and vector lattices. This method of presentation has the advantage that results for particular lattices often follow as natural specializations of results