as to directions in which things of value are to be found.

The second chapter affords a striking illustration of this tendency to introduce bits of mathematical theory without any adequate development. Here in the short space of sixteen and a half pages the reader is hustled through a discussion which touches upon Lebesgue measure, continuous groups (including the theory of group characters), and ergodic theory. It is hard to see how anyone, unless he be an expert in this particular set of subjects, can be expected to get much of anything out of this.

Similar criticisms apply to Chapter III, which deals with the theory of information. This new theory is elusive and subtle at best; and the hurried and mathematically complicated treatment given here seems to the reviewer to verge upon utter unintelligibility.

In conclusion, simple honesty and a decent regard for good workmanship compel the reviewer to remark that someone must bear the responsibility for what is truly a wretched job of proofreading. It would be hard to find another book having as high an average number of errors per page as this one has. The errors run all the way from trivialities, which are merely occasions for exasperation, to typographical errors in the equations and formulae which seriously impair the intelligibility of the exposition. Scientists, unlike philatelists, do not value documents for the errors they contain, and therefore it is to be hoped that in a second edition we shall see most of these errors corrected.

L. A. MacColl

*Topology of manifolds*. By R. L. Wilder. (American Mathematical Society Colloquium Publications, vol. 32.) New York, American Mathematical Society, 1949. 10+402 pp. $7.00.

To give an idea of the scope of this book we shall begin with a brief description of a body of theorems in the topology of the euclidean plane that are referred to as the "Schoenflies program." Let $M$ be the 2-sphere and $K$ a closed subset of $M$. If $K$ is a simple closed curve (=topological image of a 1-sphere) then $M-K$ is the union of two disjoint connected open sets $A$ and $B$ such that $K=\bar{A}\cap\bar{B}$ (Jordan Curve Theorem). It is further known that each of the sets $\bar{A}$ and $\bar{B}$ is homeomorphic to a closed disc. Converse theorems which give necessary and sufficient conditions on the set $M-K$ in order that $K$ be a simple closed curve or a Peano space (=locally connected continuum) also exist.

The objective of this book is to extend this program to higher dimensions, using homology theory as the principal tool. Thus $M$ is replaced by a generalized $n$-dimensional manifold (that is, an $n$-manifold in the sense of homology) while $K$ is in general a closed set satisfying certain connectedness and local connectedness properties. In some cases $K$ itself is a manifold of a lower dimension.

The first chapter reviews various concepts of general topology with special emphasis on connectedness. A topological characterization of the closed interval and the circumference are given. The second chapter begins with a discussion of local connectedness and then shifts to some properties of the $n$-sphere. The main theorem proved here is the Brouwer Separation Theorem which is a generalization of the Jordan Curve Theorem to the case when $M$ is an $n$-sphere and $K$ is a topological $(n-1)$-sphere. This is achieved by establishing in a very elementary manner the Alexander duality relations between the mod 2 Betti numbers of $K$ and $M-K$. Thus the reader gets his first glimpse at homology theory. Having thus proved the Jordan Curve Theorem the author proceeds to establish Schoenflies' converse theorem. Chapter three has a more detailed discussion of Peano spaces, local connectedness, uniform local connectedness, and so on. These results are applied to obtain topological characterizations of the 2-sphere, of the 2-dimensional disc, and of 2-manifolds. The fourth chapter, after some discussion of local connectedness, contains some theorems about the positional properties of a closed subset $K$ in the 2-sphere $S^2$. In particular, necessary and sufficient conditions are given which $S^2-K$ must satisfy in order that $K$ be a Peano continuum.

The first four chapters may be regarded as a modernization and slight enlargement of the Schoenflies program. However, in a sense, these chapters are still introductory, since the tools needed to carry out the program in all dimensions have not been yet introduced. This begins in chapter five where the Čech homology and cohomology theories for topological spaces are introduced. Throughout, the coefficient domain is assumed to be a field; this eliminates certain difficulties concerning so-called convergence properties. The chapter includes the duality between homology and cohomology and the theory of the cap-product. The Čech theory is "localized" in the next chapter, thus yielding the homological and co-homological theories of local connectedness. Uniform local connectedness and allied topics also are discussed. This machinery is applied in chapter seven to the study of continua. A substantial part of the Schoenflies program is thus given a significant generalization.

The full scale discussion of manifolds begins in chapter eight. Here an "$n$-dimensional generalized manifold" (abbreviated: $n$-gm) is defined as a locally compact $n$-dimensional space which is locally connected (in a sense of cohomology) in dimensions $0, \cdots, n-1$ and which at each of its points has the $n$th local Betti number equal to 1. For such manifolds the concept of orientability is defined, and, for orientable manifolds, the Poincaré duality theorem is proved. Under some additional assumptions the Alexander duality theorem comparing the Betti numbers of a closed subset of an $n$-gm and those of its complement is established. In the next chapter it is proved that a separable $n$-gm is a classical manifold for $n=1, 2$. For $n>2$ the result is no longer true. A comparison of the definition of an $n$-gm with another plausible definition is given.

The last three chapters (X–XII) are devoted to the study of positional invariants of a closed subset $K$ of an $n$-gm $M$. In the first of these chapters the complement $M-K$ is described under the assumption that $K$ is itself an $(n-1)$-gm. Conversely, imposing certain conditions on $M-K$, one can assert that $K$ is an $(n-1)$-gm. The last two chapters deal with the case when $K$ is locally connected in dimensions, $0, \cdots, k$ $(k<n)$ or is a $k$-gm. Roughly speaking "local duality theorems" are established which translate the smoothness property of $K$ at a point $x \in K$ into a property of $M-K$ at $x$. In order to achieve such a duality one has to consider concepts of accessibility, avoidability, and so on. These chapters constitute properly the generalization of the Schoenflies program to $n$ dimensions. An appendix on unsolved problems and a detailed bibliography conclude the book.

One of the chief features of the book is the unity of method. From the moment the Čech homology and cohomology are introduced (in chapter five) they are used consistently and systematically. In earlier chapters this unity is in places sacrificed for pedagogical reasons. An effort has been made not to present the reader with elaborate definitions and concepts without suitable introduction and motivation. The book is entirely self-contained and because of that a large portion of the space is devoted to the development of machinery. This machinery is sometimes quite delicate (for instance: a large number of various kinds of local connectedness are introduced, and they are compared) but this is necessitated by the fine nature of the problems considered. The results of the later chapters are almost entirely due to the author and only a small portion of them has been published before. Most of the results of the earlier chapters, although known, are not too readily available in the literature. The unification and systematic exposition of all these results is an immense task and should greatly enhance research in this part of topology.

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