in many places the usefulness and importance of this approach. In view of this it is surprising that he is reluctant to introduce and use iterated fractions (that is, sequences of general linear fractional transformations). It is true that in most cases iterated fractions can be transformed into continued fractions. However, even when this is possible, the resulting continued fraction may be considerably more involved than the iterated fraction from which it was derived. This is the case, for example, in Schur's expansion of functions bounded in the unit circle into iterated fractions. Wall uses continued fractions instead. Another instance where use of iterated fractions would have led not only to a somewhat more general result but also to a more elegant proof is in the discussion of the convergence of periodic continued fractions (§8). This could have been accomplished by the use of Schwerdtfeger's [84a] proof instead of Lane's. Finally, use of iterated fractions would have made inclusion of a discussion of the Pick-Nevanlinna interpolation problem extremely natural.

W. J. THRON

Substitutional analysis. By D. E. Rutherford. (Edinburgh University Publications, Science and Mathematics, No. 1.) Edinburgh, University Press, 1948. 11+103 pp. 25 s.

The subject matter of this book, except for the last chapter, is Young's representation theory of the symmetric group. As Young developed the methods described here, he was always thinking of the elements of the symmetric group as substitutional operators applicable, in particular, to the theory of invariants. This fact explains the title.

In the introduction the author gives a brief account of Young's life as a country clergyman whose avocation remained the development of the mathematical ideas which interested him as a student at Cambridge. Those who knew him were always impressed by his sincerity and his modesty but above all by the originality and power with which he manipulated his own complicated machinery. The present book gives a connected account of Young's methods which has long been needed. The material was scattered throughout a long series of papers and, as is not surprising, the original presentation was sometimes involved. D. E. Rutherford has simplified it materially in places, and the reader can see the significance of the various steps taken.

As indicated above the theory here described was almost incidental in Young's work; it appeared as part of a larger plan, and in this light he always considered it. Young's originality was to some extent a handicap, for it prevented him from incorporating the work of others into his scheme of things; Schur's elegant demonstration of the connection between the full linear group and the symmetric group is a case in point. This leads to perhaps the chief criticism which can be levelled against the volume under review. No attempt is made to orient Young's work either as regards its original background of invariant theory, and this is understandable, or as regards the theory of groups generally. From this latter point of view the representation theory of the symmetric group plays a unique role in that it is explicit to a degree which cannot be approached by the general theory. Having such powerful tools available, one might hope that this theory of Young's should be able to throw some light on the general theory, and this stage seems to be approaching (Nakayama, Jap. J. Math. (1940); Brauer and Robinson, Transactions of the Royal Society of Canada (1947)).

The treatment includes chapters on the calculus of permutations, the calculus of tableaux, the semi-normal representation, the orthogonal and natural representations, group characters, substitutional equations. Chapter I gives an elementary introduction to the theory of the Symmetric group S_n. In Chapter II, Young's *tableau* or *diagram* $[\alpha]\equiv[\alpha_1, \alpha_2, \cdots, \alpha_h]$ with $\alpha_i \geqq \alpha_{i+1}$ and $\alpha_1+\alpha_2+\cdots+\alpha_h=n$ is introduced and a clear account of the properties of P^α and N^α is given. Writing $E^\alpha_{rs}=\sigma_{rs}P^\alpha_s N_s$, where σ_{rs} is a substitution which transforms $[\alpha]_r$ into $[\alpha]_s$, the author obtains "Young's formula" $E^\alpha_r E^\alpha_r =\theta^\alpha E^\alpha_r$ and the remaining relations between the E's in an elegant manner.

In Chapter III the fundamental notion of a *standard* diagram in which the symbols follow an agreed order in both row and column is introduced and this leads directly to the semi-normal representation, following the method of Thrall (Duke Math. J. (1941)). The passage to the orthogonal representation is clearly explained in Chapter IV. In Young's original work the natural representation came first, and the orthogonal representation came as an impressive conclusion to a difficult piece of reading. Here some of the difficulties have been smoothed away, but the natural representation appears as an anticlimax. Though reference to it had to be included, this reviewer would have preferred that it be in an appendix. The material of §§28–31 has historical and actual value, but it serves to obscure the magnitude of Young's real achievement, the orthogonal representation.

One can sympathize with the author in the writing of §§32–36 of Chapter V. How much of the general theory of group characters should be assumed? He has chosen to assume nothing and deduces the necessary theory for the symmetric group. The sections are well written, but the uninformed reader may not realize the generality of the methods used. To the remaining paragraphs of the chapter this reviewer feels obliged to take exception. They have to do with the calculation of the characters via what has been called the Murnaghan-Nakayama recursion formula. As originally obtained by Murnaghan (Amer. J. Math. (1937)) there was little reference to Young diagrams; the irreducible components appeared with multiplicites 0, ± 1, and it remained for Nakayama (loc. cit.) to show the connection by introducing the notion of a *hook*. After the removal of the symbols $1, 2, \cdots, m$ from a standard *right* diagram $[\alpha]$, what is left can be designated by the symbol $[\alpha]-[\beta]$ and this is called a *distorted* or *skew* diagram. Just as an irreducible representation is associated with a right diagram, so a reducible representation is associated with a skew diagram. By restricting attention to the subgroup $S_m \times S_{n-m}$, the representation $[\alpha]$ of S_n breaks up into the direct sum of a number of Kronecker products of irreducible representations $[\beta]$ of S_m and reducible representation $[\alpha]-[\beta]$ of S_{n-m}. Thought of from this point of view the multiplicites 0, ± 1 are characters of cycles of length $n-m$ in the skew representation $[\alpha]-[\beta]$ (Robinson, Amer. J. Math. (1947) and (1948)). In the last chapter the "substitutional" aspect of the theory is illustrated. Some words of explanation here would have helped the reader to see the usefulness of the results obtained.

To sum up: the book provides a long overdue account of Young's representation theory of the symmetric group. Many of Young's complicated proofs have been simplified through a free use of mathematical induction. The printing is excellent, and no errors have been detected. The Edinburgh University Press is to be congratulated on such a handsome first volume.

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