BOOK REVIEWS

Les méthodes modernes du calcul des probabilités et leur application au problème de la turbulence. By J. Bass. (Groupement français pour le développement des Recherches Aéronautiques, rapport technique no. 28.) Paris, 1946, 241 pp.

This is a technical report on methods of probability theory and their application to the problem of turbulence. It completely fulfills its purpose which is to give to the physicists a thorough picture of the theories involved as they stood in 1944. It contains also indications of the contributions of the author and of other French mathematicians in 1945 but nothing about the contributions of Kolmogoroff and his school, unknown in France at the date of publication (1946). The author does not intend to give complete and rigorous proofs but mostly indicates the lines along which such proofs have been obtained.

The first part deals with probability theory. In the first four chapters the author treats stochastic processes with independent increments and Markoff chains. Chaps. V to VIII are concerned with random functions: stochastic differentiation and integration, and the stationary case.

The second part deals with the statistical theories of turbulence. It starts with the Navier equations and the Reynolds theory, examines the Taylor theory, then the Dedebant and Wehrlé theory, and finishes with the spectrum and the levels of perturbation.

M. LOÈVE

Rings and ideals. By N. H. McCoy. (Carus Mathematical Monographs, no. 8.) Buffalo, Mathematical Association of America, 1948. 12+216 pp.

A ring is a set of elements which forms an abelian group under addition and is closed under multiplication which is associative and distributive. Postulates for a ring differing only slightly from the above were given by Fraenkel in 1914.

The modern theory of linear associative algebras dates from Wedderburn's thesis of 1907. A linear associative algebra is a ring which is specialized by possessing a finite basis relative to a field. This basis plays an essential part in Wedderburn's treatment and in most of the work which has followed it. Several attempts have been made, one by Wedderburn himself, to extend the theory to algebras with an infinite basis. Probably the most successful generalization was made by Artin in 1927 by ignoring the basis and extending the main features of the Wedderburn structure theory to rings satisfying the chain conditions. Since then the theory of rings has developed steadily through the researches of many writers such as Noether, Krull, Köthe, von Neumann, Albert, Brauer, Jacobson, and McCoy, to mention only a few. That so many detailed theorems can be obtained from such general hypotheses is indeed remarkable, and attests the power of the abstract analysis.

The concept of ideal had its genesis in the study of algebraic numbers, where it was introduced by Kummer and Dedekind. Wedderburn based his theory of algebras on the concept of invariant subalgebra but later writers, recognizing the similarity of the two concepts, have used the word ideal in both connections.

The book under review is an excellent introduction to this modern branch of algebra. Seldom does one have the pleasure of reading a book which has been written with so much care and expository skill. The difficulties have been so carefully anticipated that the reader is apt to get a false impression of the simplicity of the material.

There are nine chapters in the course of which the pace is gradually increased. The first two involve well known material freshly presented. In Chapter III on *Ideals and homomorphisms* some of the fundamental properties of finite fields are very simply obtained. Chapter IV deals with the imbedding of one ring in another, particularly with quotient rings. The plot begins to thicken in Chapter V, *Prime ideals in commutative rings*. Use is made of Zorn's Lemma, which McCoy calls the Maximum Principle. It is a form of Zermelo's axiom.

Chapter VI is entitled *Direct and subdirect sums*. This is material in the development of which the author has taken part. A result startlingly similar to the Wedderburn decomposition theorem for linear algebras holds in a commutative ring R. If $\mathfrak{y} \neq R$ is the Jacobson radical, $R/\mathfrak{y}$ is isomorphic to a subdirect sum of fields.

Chapter VII is devoted to Boolean rings and the algebra of logic, and their generalizations, the p-rings. Chapter VIII is concerned with rings of matrices whose elements belong to an arbitrary ring R. For the introduction of determinants it becomes necessary to assume that R is commutative with more than one element, and that the unit element 1 is present. With these assumptions most of the theory of determinants and systems of equations carry over, but not without serious modifications in the usual theory. The extension of the concept of rank is particularly ingenious, but the reviewer will refrain from revealing the plot. The reader may find it on page 159. The author's own researches on the characteristic ideal and the null ideal of a matrix are developed in this chapter.

The concluding chapter is entitled *Further theory of ideals and commutative rings* and is fairly concentrated. It is concerned with Noetherian rings and algebraic manifolds and is designed, probably, to leave the reader in a more humble frame of mind.

It is surprising that this book and the one by Jacobson (*The theory of rings*, Mathematical Surveys, no. 2, New York, 1943) overlap so little. Doubtless McCoy planned it that way. Jacobson had more space at his disposal and his book has not been supplanted for reference purposes. But the McCoy book has many novel points of view and some more recent material, and as an introduction to the powerful and highly abstract method of thinking which now characterizes modern algebra, it is a gem.

C. C. MacDuffee

Mathematical analysis of binocular vision. By R. K. Luneburg. Princeton University Press, 1947. 6+104 pp. $2.50.

An attempt is made in this study to derive a metric for the psychological space of binocular vision. It is first shown that the recognition of greater and smaller and of greater and smaller contrast uniquely determines the psychometric coordination of numbers to sensation within the limits of a linear transformation. In order to proceed there is then introduced a rather strong hypothesis which is suggested by some experimental observations.

Let an observer first view a point P with head fixed. For this situation a convenient set of coordinates is γ, ϕ, θ, where γ is the angle of convergence (angle LPR, L and R representing the eyes), ϕ is a lateral angular deviation ($PLR/2-PRL/2$), and θ is the ang lar elevation of the plane PRL from the horizontal plane. Next let the observer be permitted to rotate his head about a vertical axis so that the eyes converge symmetrically on P. For this situation another set of coordinates γ^*, ϕ^*, θ^* is introduced. The corresponding angles are very similarly defined. In the transformation from cartesian coordinates to these starred coordinates the distance d' between the line through the eyes and the axis of rotation enters as a parameter.[1]

Now suppose we have two different configurations of object points, the first being viewed with fixed head, the second being viewed with rotating head. If there is a correspondence between the two such that γ for the first equals γ^* for the second and similarly $\phi=\phi^*$, $\theta=\theta^*$,

[1] It would seem that d' could be experimentally made to take on values from zero, or even less, to many times the normal value. If so it would be of interest to determine whether or not the hypothesis would hold when the effect is accentuated by choosing extreme values of d'.