THE ANNUAL MEETING OF THE SOCIETY

The fifty-fifth Annual Meeting of the American Mathematical Society was held at the Ohio State University, Columbus, Ohio, Tuesday to Thursday, December 28–30, 1948, in conjunction with the Annual Meeting of the Association for Symbolic Logic, the Annual Meeting of the Mathematical Association of America, and the Christmas Conference of the National Council of Teachers of Mathematics.

Over 500 people registered for the meeting, among whom were the following 441 members of the Society.

A. W. Adkisson, J. E. Adney, R. P. Agnew, E. J. Akutowicz, L. U. Albers, A. A. Albert, C. B. Allendoerfer, Warren Ambrose, B. A. Amirà, E. W. Anderson, R. D. Anderson, R. V. Andree, Nachman Aronszajn, Max Astrachan, M. C. Ayer, Frank Ayres, W. L. Ayres, Reinhold Baer, F. R. Bamforth, G. M. Bareis, W. E. Barnes, I. A. Barnett, A. F. Bausch, H. M. Beatty, J. C. Bell, J. H. Bell, J. L. Bell, A. A. Bennett, Theodore Bennett, W. D. Berg, Stefan Bergman, Felix Bernstein, William Betz, R. H. Bing, Z. W. Birnbaum, A. H. Black, C. J. Blackall, H. D. Block, Henry Blumberg, L. M. Blumenthal, R. P. Boas, H. W. Bode, W. M. Boothby, J. G. Bowker, M. G. Boyce, G. F. Bradfield, H. J. Bradley, A. T. Brauer, Richard Brauer, H. E. Bray, H. W. Brinkmann, J. R. Britton, Foster Brooks, R. S. Burington, Herbert Busemann, L. E. Bush, Jewell H. Bushey, W. H. Bussey, S. S. Cairns, R. H. Cameron, C. C. Camp, A. B. Carson, K. C. Cartwright, C. R. Cassity, J. W. Cell, Lamberto Cesari, K. Chandrasekharan, Harold Chatland, Y. W. Chen, Alonzo Church, Edmund Churchill, R. V. Churchill, W. G. Clark, J. A. Clarkson, M. D. Clement, L. W. Cohen, J. A. Cooley, N. A. Court, R. R. Coveyou, W. H. H. Cowles, V. F. Cowling, E. H. Crisler, J. S. Cronin, J. C. Currie, H. B. Curry, D. A. Darling, L. A. V. DeCleene, C. H. Denbow, R. F. Deniston, A. H. Diamond, R. P. Dilworth, C. L. Dolph, M. D. Donsker, J. L. Doob, H. L. Dorwart, C. H. Dowker, W. C. Doyle, Arnold Dresden, Melvin Dresher, R. J. Duffin, W. L. Duren, Aryeh Dvoretzky, E. L. Eagle, E. D. Eaves, Samuel Eilenberg, H. M. Elliott, J. H. Engel, H. P. Evans, G. M. Ewing, A. B. Farnell, Herbert Federer, William Feller, F. A. Ficken, N. J. Fine, C. D. Firestone, M. P. Fobes, L. R. Ford, G. E. Forsythe, Tomlinson Fort, R. H. Fox, J. S. Frame, Orrin Frink, W. H. J. Fuchs, R. E. Fullerton, B. E. Gatewood, H. M. Gehman, H. H. Germond, F. J. Gerst, David Gilbarg, B. P. Gill, Leonard Gillman, Wallace Givens, A. M. Gleason, E. L. Godfrey, Casper Goffman, Michael Goldberg, Michael Golomb, A. W. Goodman, R. E. Goodman, V. G. Gorciu, Saul Gorn, M. J. Gottlieb, S. H. Gould, L. M. Graves, J. B. Greeley, J. W. Green, L. J. Green, L. Z. Greene, V. G. Grove, P. E. Guenther, William Gustin, S. W. Hahn, Elizabeth Hahnemann, C. B. Hailperin, Franklin Haimo, Marshall Hall, P. R. Halmos, Frank Harary, O. G. Harrold, W. L. Hart, E. E. Haskins, J. O. Hassler, G. E. Hay, C. T. Hazard, A. E. Heins, R. G. Helsel, Fritz Herzog, Edwin Hewitt, E. H. C. Hildebrandt, T. H. Hildebrandt, Einar Hille, J. J. L. Hinrichsen, A. J. Hoffman, D. L. Holl, Carl Holtom, E. M. Hove, J. A. Hratz, R. C. Huffer, Ralph Hull, P. M. Hummel, W. R. Hutcherson, L. C. Hutchinson, M. A. Hyman, R. M. Iwanowski, A. W. Jacobson, R. L. Jeffery, Herbert Jehle, E. D. Jenkins, Walter Jennings, A. W. Jones, B. W. Jones, M. E. Jones, P. S. Jones, Bjarni Jónsson, Mark Kac, Shizuo Kakutani, L. H. Kanter, Irving Kaplansky,

H. T. Karnes, Chosaburo Kato, M. E. Kellar, M. W. Keller, J. L. Kelley, A. J. Kempner, J. R. F. Kent, D. E. Kibbey, W. M. Kincaid, J. R. Kinney, S. C. Kleene, J. R. Kline, P. A. Knedler, L. C. Knight, L. A. Knowler, D. M. Krabill, M. S. Kramer, H. W. Kuhn, O. E. Lancaster, R. E. Langer, G. A. Larew, E. H. Larguier, H. D. Larsen, C. G. Latimer, V. V. Latshaw, J. S. Leech, Joseph Lehner, F. A. Lewis, B. J. Lockhart, Charles Loewner, E. R. Lorch, L. L. Lowenstein, Eugene Lukacs, N. H. McCoy, S. W. McCuskey, W. C. McDaniel, J. C. C. McKinsey, E. J. McShane, C. C. MacDuffee, G. R. MacLane, Saunders MacLane, Ingo Maddaus, Szolem Mandelbrojt, H. B. Mann, C. G. Maple, Morris Marden, R. H. Marquis, George Marsaglia, R. M. Martin, W. T. Martin, M. E. Martinson, W. S. Massey, F. I. Mautner, J. R. Mayor, P. E. Meadows, A. E. Meder, C. W. Mendel, G. M. Merriman, E. L. Mickelson, E. J. Mickle, C. C. Miesse, H. J. Miles, D. D. Miller, Frederic H. Miller, L. H. Miller, W. E. Milne, H. J. Miser, E. E. Moise, C. N. Moore, F. R. Morris, D. C. Morrow, Marston Morse, G. D. Mostow, W. B. Moye, J. R. Musselman, D. M. Nead, Zeev Nehari, W. J. Nemerever, C. V. Newsom, O. M. Nikodým, E. P. Northrop, I. L. Novak, F. S. Nowlan, C. O. Oakley, R. E. O'Connor, F. C. Ogg, Rufus Oldenburger, L. F. Ollmann, E. J. Olson, Paul Olum, Morris Ostrofsky, E. R. Ott, F. W. Owens, H. B. Owens, Gordon Pall, W. V. Parker, H. C. Parrish, Philip Peak, S. E. Pence, P. M. Pepper, Mary Pettus, Everett Pitcher, Harry Polachek, J. C. Polley, J. W. Ponds, G. B. Price, F. M. Pulliam, Tibor Rado, E. D. Rainville, J. F. Randolph, S. E. Rasor, M. O. Reade, L. M. Reagan, O. W. Rechard, Mina Rees, P. V. Reichelderfer, W. P. Reid, Eric Reissner, Daniel Resch, M. M. Resnikoff, C. N. Reynolds, D. E. Richmond, R. F. Rinehart, L. A. Ringenberg, R. A. Roberts, L. V. Robinson, V. N. Robinson, W. J. Robinson, L. D. Rodabaugh, T. G. Room, P. C. Rosenbloom, Arthur Rosenthal, M. F. Rosskopf, S. A. Rowland, L. R. Rubashkin, C. H. Rust, W. A. Rutledge, Raphael Salem, Charles Saltzer, R. G. Sanger, L. A. Santaló, S. W. Saunders, A. C. Schaeffer, Robert Schatten, S. A. Schelkunoff, M. M. Schiffer, E. R. Schneckenberger, K. C. Schraut, H. M. Schwartz, G. E. Schweigert, C. E. Sealander, C. L. Seeback, I. E. Segal, Wladimir Seidel, M. E. Shanks, H. C. Shaub, Seymour Sherman, S. S. Shü, C. N. Shuster, Edward Silverman, L. L. Silverman, F. C. Smith, R. E. Smith, J. L. Snell, W. S. Snyder, Andrew Sobczyk, T. H. Southard, D. C. Spencer, C. E. Springer, George Springer, G. W. Starcher, E. P. Starke, N. E. Steenrod, H. E. Stelson, Rothwell Stephens, Guy Stevenson, B. M. Stewart, R. W. Stokes, D. M. Stone, M. H. Stone, R. B. Stone, E. B. Stouffer, A. C. Sugar, E. G. Swafford, Otto Szasz, Alfred Tarski, J. S. Taylor, M. E. Taylor, William Charles Taylor, William Clare Taylor, H. P. Thielman, L. O. Thompson, R. M. Thrall, G. L. Tiller, H. E. Tinnappel, Leonard Tornheim, M. M. Torrey, H. M. Trent, Deonisie Trifan, W. J. Trjitzinski, C. A. Truesdell, A. W. Tucker, A. R. Turquette, J. L. Ullman, Gilbert Ulmer, E. P. Vance, Henry Van Engen, H. E. Vansant, H. E. Vaughan, R. W. Wagner, G. L. Walker, R. J. Walker, S. E. Walkley, J. L. Walsh, J. B. Walton, W. R. Wasow, André Weil, Alexander Weinstein, B. A. Welch, F. P. Welch, E. T. Welmers, I. W. Welmers, F. J. Weyl, Hermann Weyl, George Whaples, E. A. Whitman, P. M. Whitman, D. R. Whitney, Hassler Whitney, G. T. Whyburn, R. B. Wildermuth, F. B. Wiley, R. B. Wiley, S. S. Wilks, W. L. Williams, C. O. Williamson, R. L. Wilson, J. E. Yarnelle, J. W. T. Youngs, Arthur Zeichner, Daniel Zelinsky, M. A. Zorn, Antoni Zygmund.

The twenty-second Josiah Willard Gibbs Lecture, entitled *Ramifications, old and new, of the eigenvalue problem*, was delivered by Professor Hermann Weyl of the Institute for Advanced Study on Tuesday, December 28. Professor Einar Hille, President of the American Mathematical Society, was the presiding officer.

The Committee to Select Hour Speakers for Annual and Summer Meetings invited three speakers. Professor Mark Kac of Cornell University lectured on *Probability methods in some problems of analysis and theory of numbers* on Tuesday, December 28. Professor Antoni Zygmund presided. On Wednesday, December 29, Professor J. L. Walsh presided over two lectures on area theory. Professor A. S. Besicovitch of Cambridge University and the University of Pennsylvania spoke on *Parametric surfaces*; Professor Lamberto Cesari of the University of Bologna, the Institute for Advanced Study, and the Ohio State University lectured on the topic *Area and representation of surfaces.*

The Annual Business Meeting and Election of Officers was held on Wednesday, December 29. Details of proceedings are reported in the sequel. The Bôcher Memorial Prize was awarded jointly to Professors A. C. Schaeffer of Purdue University and D. C. Spencer of Stanford University for their papers on *Coefficients of schlicht functions* published in Duke Math. J. vol. 10 (1943) pp. 611–635; vol. 12 (1945) pp. 107–125; Proc. Nat. Acad. Sci. U.S.A. vol. 32 (1946) pp. 114–116. Professor Schaeffer gave a brief summary of the papers.

After the lecture by Professor Schaeffer, Professor Einar Hille of Yale University, President of the American Mathematical Society, delivered the Presidential Address entitled *Lie theory of semi-groups of linear transformations.* Professor J. L. Walsh, President Elect, was the presiding officer.

The organized social events provided for the Society consisted of:

A reception and tea on Tuesday afternoon in the Faculty Club.

A recital on Wednesday evening by a duo-piano team and a vocalist from the School of Music.

A motion picture on Wednesday afternoon and a tea for the ladies on Thursday afternoon given by the Math Circle, a group consisting of ladies who are connected with the Department of Mathematics as staff members or wives of staff members.

A dinner on Thursday night in the Mach-Canfield dining hall. Professor R. E. Langer acted as toastmaster, and there were speeches by Vice-President Harlan Hatcher (The Ohio State University), Professor Saunders MacLane and Professor Marston Morse. Professor R. P. Dilworth read a resolution of thanks to The Ohio State University for their hospitality, highly commending the Local Committee on Arrangements, Professor R. G. Helsel, Chairman, for the obvious attention to detail which characterized the meeting.

At the meeting of the Board of Trustees at 6:00 P.M. on December 28, 1948, there was no quorum present and the Board adjourned to January 15, 1949.

The Council met at 9:30 P.M. on December 28.

The Secretary announced the election of the following fifty-two persons to ordinary membership in the Society:

Mr. Andrew Norwood Aheart, West Virginia State College;
Mr. Bernard Altshuler, New York University;
Dr. Frederick Dewey Bennett, Ballistics Research Laboratory, Aberdeen Proving Ground, Md.;
Professor Rubens Betelman, Curitiba, Parana, Brazil;
Professor Thomas Alton Bickerstaff, University of Mississippi;
Mr. Donald Watson Blackett, Princeton University;
Miss Mildred Ellen Blackman, St. Ambrose College, Davenport, Iowa;
Mr. Isaac Edward Block, Harvard University;
Mr. Eleazer Bromberg, Reeves Instrument Corporation, New York City;
Mr. Howard Henry Burt, General Fireproofing Company, Youngstown, Ohio;
Mr. Kenneth Vincent Casey, Brown University;
Brother Damian Connolly, LaSalle College, Philadelphia, Pa.;
Mr. Stanley Fifer, Reeves Instrument Corporation, New York City;
Mr. Donald Mandel Friedlen, Illinois Institute of Technology;
Mr. Richard Francis Gabriel, St. Francis College, Brooklyn, N. Y.;
Mr. Paul Guy Galentine, Jr., Willow Run Village, Michigan;
Mr. Irving M. Garfunkel, Aero Research Center, Willow Run Airport, Ypsilanti, Michigan;
Dr. George Alvin Garrett, Carbide and Carbon Chemicals Corporation, Oak Ridge, Tenn.;
Mrs. Kathe Solis-Cohen Jacoby, Philadelphia, Pa.;
Mr. Leonard Guy Jones, University of Oregon;
Mr. Kenneth Edward Kain, Belleville, Ill.;
Miss Jane F. Kiefer, Columbia University;
Professor Pedro Laborde-Montaner, New York University;
Professor Werner Walter Leutert, University of Maryland;
Mr. Julius Lieblein, Statistical Engineering Laboratory, National Bureau of Standards, Washington, D. C.;
Mr. Harold Adrian Linstone, Aerophysics Laboratory, North American Aviation, Inc., Downey, Calif.;
Professor Henry Lisman, Yeshiva University;
Professor Everett William McClane, DePaul University;
Mr. John A. McKee, Wilkes College, Wilkes Barre, Pa.;
Mr. Robert Colegrove Meacham, Brown University;
Mr. Paul Alfred Moser, Coalinga Junior College, Coalinga, Calif.;
Professor Sam Naiditch, Chemistry Dept., Duquesne University;
Professor Paul Nastucoff, University of Notre Dame;
Mr. William Joseph Nemerever, University of Michigan;
Professor Loyd Clinton Oleson, Doane College, Crete, Neb.;
Mr. Alexander Orden, Massachusetts Institute of Technology;
Professor Mary Pettus, Lander College, Greenwood, S. C.;
Mr. Constantinos George Plithides, Columbia University;

Mr. Murray L. Polk, Naval Shipyard, Brooklyn, N. Y.;
Dr. Joseph Ehrman Pryor, Harding College, Searcy, Ark.;
Mr. Edward Rayher, Bergen Junior College, Teaneck, N. J.;
Mr. Lionel Israel Rebhun, Brooklyn, N. Y.;
Mr. Paulo Ribenboim, Rio de Janeiro, Brazil;
Mr. Alfredo Milton Vila-Flor Santos, Salvador, Bahia, Brazil;
Professor Gobind Ram Seth, Iowa State College of Agriculture and Mechanic Arts;
Professor Donald Arthur Steele, Fordham University;
Miss Herta Taussig, Hollins College, Hollins College, Va.;
Mr. John C. Thompson, State Teachers College, Dickinson, N. D.;
Professor Billie Braden Townsend, Louisiana State University;
Dr. Horace Maynard Trent, Naval Research Laboratory, Washington, D. C.;
Mr. Daniel Hobson Wagner, Brown University;
Mr. Stanley Simon Walters, University of California at Los Angeles.

It was reported that the following one hundred twenty-seven persons had been elected to membership on nomination of institutional members as indicated:

University of Alabama: Mr. Louis Jaffe, Mrs. Ayrlene McGahey Jones;
Brooklyn College: Mr. Melvin Hausner;
Brown University: Professor Erastus Henry Lee and Mr. Elliot Samuel Wolk;
California Institute of Technology: Messrs. Robert James Diamond and Thomas George Macfarlane;
University of California: Messrs. James Michael Gardner Fell, Steven Lyle Jamison, Ralph Mortimer Lakness, Frederick Burtis Thompson, Robert Lawson Vaught, Donald Dines Wall, and Philip Starr Wolfe;
University of California at Los Angeles: Messrs. Eugene Howard Jacobs and Douglas Houston Moore;
University of Chicago: Messrs. Robert Gardner Bartle, Nathan Joseph Divinsky, Henry Abel Dye, Richard Vincent Kadison, Henry William Oliver, Charles Morton Price, Isadore Manual Singer, Grayson Letcher Tucker, Jr., Louis Max Weiner, and Edgar Samuel Williams;
City College, New York City: Messrs. David Finkelstein, Donald J. Newman, Harold Seymour Shapiro, and Robert Angelo Spinelli;
Columbia University: Messrs. Calvin Creston Elgot, Cornelius Wilde Langley, Irwin Mann, John Rausen, Hsien Chang Tsang, Hermann Valentin Waldinger, Jerome Harris Weiner, and David Wellinger;
Cornell University: Messrs. Wallace Edward Barnes, Ernest Slavko Elyash, George Seifert, and Murray R. Spiegel;
Duke University: Messrs. James Richard Garrett and Joseph Andrew Silva;
Illinois Institute of Technology: Mr. Howard Lahman Arnould, Miss Maria Davidson, Messrs. Pasquale Porcelli and Albert Soglin;
University of Illinois: Messrs. Paul F. Conrad, William Alvin Howard, George Roger Livesay, Shu-Teh Chen Moy, David Arthur Page, and William G. Rosen, Miss Jean Elaine Sammet, Messrs. Judson Samderson, Jr., James Harold Turnock, Jr., and John Eldon Whitesitt;
Indiana University: Mr. James Burton Serrin, Jr.;
Institute for Advanced Study: Professor Lamberto Cesari, Messrs. John George Kemeny, Leopold Alexander Pars, and Kollagunta Gopalaiyer Ramanathan, Professors Kurt Werner Reidemeister, and Herbert K. J. Seifert;

Iowa State College of Agriculture and Mechanic Arts: Mr. Robert N. Goss;
State University of Iowa: Mr. William Henry Marlow;
Johns Hopkins University: Messrs. Peter Henry Berning, Charles Henry Murphy, Jr., and Malcolm William Oliphant, Miss Mary McCulloch Templeton, Mr. Kenneth G. Wolfson;
University of Kansas: Miss Margaret Marie Pihlblad;
Lehigh University: Messrs. Henry Albert Seebald and Michael Tikson;
University of Maryland: Messrs. Edward Crownshield Higgison and William Russell Thickstun, Jr.;
Massachusetts Institute of Technology: Messrs. Frederick Sheppard Holt, Ernest Ray Keown, and Barrett O'Neill;
Michigan State College: Mr. Charles Hall Kraft;
University of Michigan: Messrs. Charles Francis Briggs, Morton Landers Curtis, Howard Raiffa, Ralph Leland Shively and William Kay Smith;
University of Minnesota: Mr. Robert H. Scherer;
University of Missouri: Mr. William Lee Stamey;
Northwestern University: Mr. Donald Y. Barrer;
Ohio State University: Messrs. Ray Edward Kidder, Norman Levine, Robert Vernon Mendenhall, and William Mackie Myers, Jr.;
Oklahoma Agricultural and Mechanical College: Mr. Franklin Arno Graybill;
College of St. Thomas: Mr. Ephriam Joseph Vitoff;
Stanford University: Mr. Halsey Lawrence Royden, Jr.;
Swarthmore College: Mr. Robert Zane Norman;
Syracuse University: Mr. Robert S. Finn;
Texas Technological College: Professor Emmett A. Hazelwood;
University of Texas: Messrs. Billy Joe Ball, John Herbert Barrett, C. Edmund Burgess, and George Copp;
University of Toronto: Messrs. Thomas Edward Hull and John Albert Rottenberg;
University of Virginia: Mr. Henry Francis DeFrancesco;
University of Washington: Messrs. Yeh Mo and Ernest Carl Schlesinger;
Wellesley College: Miss Zung-nyi Loh;
Wesleyan University: Messrs. James Louis Dolby and Robert Thomas Mathews;
Williams College: Professor Chester Henry Gordon;
University of Wisconsin: Mr. Henry Arthur Friedman, Miss Rachel Sherman Hodges, Messrs. Wallace Eugene Johnson, Benjamin Evans Mitchell, Harold Theodore Slaby, Kennan Tayler Smith, and Daniel Sokolowsky, Miss Margaret Alice Waugy, Dr. Chia-shuen Yih, Messrs. Frederick Harris Young and Allen Douglas Ziebur;
Yale University: Messrs. Robert Roland Christian, Charles Whittlesey Curtis, Eugene (Victor) Schonkman and Alfred Burton Willcox.

The Secretary announced that the following had been admitted to the Society in accordance with reciprocity agreements with various mathematical organizations: French Mathematical Society: Professor Ferran Sunyer Balaguer, University of Barcelona; Professor Ratip Berker, Université Technique d'Istanbul; Professor Wenceslas Jardetzky, University of Graz; London Mathematical Society: Professor Hugh Patrick Mulholland, American University, Beirut, Lebanon; Professor Thomas Gerald Room, University of Sydney;

Swiss Mathematical Society: Dr. Hugo Ribeiro, University of California; Professor Ernest Trost, Technicum Winterthur, Zurich; Unione Matematica Italiana: Dr. Iacopo Barsotti, Princeton University.

It was reported that the Council had voted by mail to accept the recommendation of the Committee on the Role of the Society in Mathematical Publication that the Society publish three volumes of the Transactions in 1949, each volume to consist of approximately 500 pages and the subscription price to be $6.00 list per volume (members of the Society to receive the usual twenty-five per cent discount). The Board of Trustees also voted to approve this recommendation.

The Council had also voted by mail to accept the invitation of the University of Pennsylvania to meet in Philadelphia on April 29–30, 1949, this meeting to be in place of that previously scheduled by the Council for New York City.

The Secretary is pleased to report at this time that the ordinary membership of the Society is now 3811, including 333 nominees of institutional members and 53 life members. There are also 105 institutional members. The total attendance at all meetings in 1948 was 2424; the number of papers read was 610; there were 16 hour addresses, 6 addresses and 11 papers at the Applied Mathematics Symposium, 1 Gibbs Lecture, 4 Colloquium Lectures, and 1 Retiring Presidential Address; the number of members attending at least one meeting was 1432.

The following appointments by President Einar Hille of representatives of the Society were reported: Professor J. N. Michie at inauguration of James P. Cornette as President of West Texas State College on October 2, 1948; Professor E. B. Allen at inauguration of Jess Harrison Davis as President of Thomas S. Clarkson Memorial College of Technology on October 8, 1948; Professor J. R. Kline at installation of Dwight David Eisenhower as President of Columbia University on October 12, 1948; Professor C. N. Moore at Seventy-Fifth Anniversary Celebration of The Ohio State University on October 14–15, 1948; Professor T. R. Hollcroft at inauguration of Alan Willard Brown as President of Hobart and William Smith Colleges on October 22–23, 1948; Professor J. B. Rosenbach at inauguration of William Granger Ryan as President of Seton Hill College on November 11, 1948.

The following additional appointments by the President were reported: Professors J. H. Roberts (Chairman), F. G. Dressel, Tomlinson Fort, W. M. Whyburn as Committee on Arrangements for meeting at Duke University on April 1–2, 1949; Professors G. W. Smith

(Chairman), Wealthy Babcock, Paul Eberhart, G. B. Price, R. G. Sanger, J. W. T. Youngs as Committee on Arrangements for meeting at University of Kansas on April 29–30, 1949; Professor R. V. Churchill (Chairman), Dean Walter Bartky, Professors E. L. Eriksen, G. E. Hay, D. L. Holl, J. W. T. Youngs as Committee on Arrangements for Third Annual Symposium in Applied Mathematics at University of Michigan in June, 1949; Professors J. R. Kline (Chairman), E. F. Beckenbach, G. T. Whyburn as a committee to nominate representatives of Society on Policy Committee for Mathematics; Professors C. C. MacDuffee (Chairman), A. A. Albert, Emil Artin as committee on award of Cole Prize in Algebra, to be awarded at 1949 Annual Meeting; Professors S. S. Cairns (Chairman), J. R. Kline, P. A. Smith as committee to study questions raised by Committee on Revision of By-Laws; Dr. E. R. Kolchin, Professors C. E. Sealander and Walter Strodt as tellers for 1948 annual election; Professor G. A. Hedlund as a member of the Committee to Select Hour Speakers for Summer and Annual Meetings for period 1949–1950 (committee now consists of Professors J. R. Kline, Chairman, G. A. Hedlund, and T. H. Hildebrandt); Professor D. V. Widder as a member of the Committee to Select Hour Speakers for Eastern Sectional Meetings for period 1949–1950 (committee now consists of Professors T. R. Hollcroft, Chairman, Deane Montgomery, and D. V. Widder); Professor Ralph Hull as a member of the Committee to Select Hour Speakers for Western Sectional Meetings for period 1949–1950 (committee now consists of Professors J. W. T. Youngs, Chairman, Marshall Hall, and Ralph Hull); Professor E. F. Beckenbach as a member of the Committee to Select Hour Speakers for Far Western Sectional Meetings for period 1949–1950 and Professor D. C. Spencer as a member for 1949 (committee now consists of Professors J. W. Green, Chairman, E. F. Beckenbach, and D. C. Spencer); Professor J. M. Thomas as a member of Committee on Printing Contracts for period 1949–1951 (committee now consists of Professor C. J. Rees, Chairman, Dean M. H. Ingraham, and Professor J. M. Thomas); Professor Richard Courant as a member of Committee on Visiting Lectureship for period 1949–1951 (committee now consists of Professors Hassler Whitney, Chairman, Richard Courant, and Dean R. G. D. Richardson); Professors William Prager and Eric Reissner as members of Committee on Applied Mathematics for period 1949–1951 (committee now consists of Dean Walter Bartky, Chairman, Professors R. V. Churchill, G. C. Evans, John von Neumann, William Prager, and Eric Reissner).

It was reported that Professor Deane Montgomery of the Institute for Advanced Study had accepted the invitation to deliver a series of Colloquium Lectures at the first Summer Meeting following that of 1949 and that the title of the Lectures would be *The topology of groups.*

At the annual election which closed on December 29, and at which 801 votes were cast, the following officers were elected:

President, Professor J. L. Walsh.

Vice President, Professor W. T. Martin.

Secretary, Professor J. R. Kline.

Associate Secretary, Professor T. R. Hollcroft.

Treasurer, Dean A. E. Meder.

Member of the Editorial Committee of the Bulletin, Professor Deane Montgomery.

Member of the Editorial Committee of the Transactions, Professor Saunders MacLane.

Member of the Editorial Committee of the Colloquium Publications, Professor Einar Hille.

Member of the Editorial Committee of Mathematical Reviews, Professor Hassler Whitney.

Member of the Editorial Committee of Mathematical Surveys, Professor A. W. Tucker.

Representative on the Editorial Board of the American Journal of Mathematics, Professor L. M. Graves.

Members at large of the Council, Professors Emil Artin, Herbert Busemann, M. H. Heins, G. W. Mackey, and L. C. Young.

Members of the Board of Trustees, Professors T. H. Hildebrandt, W. R. Longley, Dean R. G. D. Richardson, Professors P. A. Smith and G. T. Whyburn.

The Council voted to approve June 14–16, 1949, as the dates for the Third Annual Symposium in Applied Mathematics to be held at the University of Michigan, the topic for the Symposium to be *Elasticity*. Dates of other meetings during 1949 were set as follows: August 30–September 2 at University of Colorado; October 29 in New York City; November 26 in Pasadena, California; December 27–29 in New York City.

The following resolution on the retirement of Professor B. P. Gill as Treasurer of the Society was adopted by the Council:

It is with profound regret that the Council and the Board of Trustees have accepted the decision of Professor Bennington P. Gill to retire from the Treasurership of the American Mathematical Society on December 31, 1948. His careful study of modern accounting practices to discover the methods best adapted to the business of the Society has resulted in reports which have brought high praise from the auditors.

As Secretary of the Board of Trustees he has presented matters to be considered in a form which greatly facilitated the transaction of the business in hand. During the past ten years there has been a great increase in the labor involved in caring for the funds of the Society and in handling the business connected with its publications. For his efficiency and devotion in performing the tasks of his office Professor Gill deserves the appreciation and thanks of all members of the Society.

In an appendix to this report are excerpts from the report of the Treasurer for the fiscal year 1948 as verified by the auditors. A copy of the complete report will be sent, on request, to any member of the Society.

The American Journal of Mathematics, which is a joint enterprise of the American Mathematical Society and The Johns Hopkins University, reported that it had published 908 pages in 1948. The Society is also giving a subvention to the Canadian Journal of Mathematics, the first number of which was on display at the registration desk.

The Librarian reported the following additions to the Library: 122 volumes of periodicals, 68 books, 94 pamphlets (including 44 dissertations).

Certain invitations to give addresses in 1949 were announced: Professor Stefan Bergman for the February meeting in New York City; Professor J. H. Roberts for the April meeting at Duke University; Professors G. W. Mackey and Kurt Reidemeister for the April meeting in Philadelphia.

The Bulletin Editorial Committee reported the increasing backlog of manuscripts accepted for publication in the Bulletin which cannot be published promptly as was formerly the case. There was considerable discussion in the Council of the serious situation confronting the Society with respect to publication and the Council voted to authorize the appointment of a special committee to consider the emergency situation. Professors R. H. Bing and I. M. Sheffer were reported as new Assistant Editors for the Bulletin.

The Transactions Editorial Committee reported that a total of 1196 pages had been published in the 1948 volumes. Plans for the publication of three volumes in 1949 were reported: the first volume will consist of the January, March and May issues; the second volume will be an extra volume of 500 pages, to be bound in two parts; the third volume will be a two number volume. Professor Garrett Birkhoff was reported as a new Associate Editor and Professor G. T. Whyburn as Managing Editor.

The Mathematical Reviews Editorial Committee reported that the amount of material received had continued to increase, that 638 pages of reviews had been published in 1948 as against 616 for 1947 and 540 for 1946. During 1948 the Indian Mathematical Society was added to the list of sponsors of Mathematical Reviews. The subscription list, as of December 1, 1948, was 1,966.

Professors J. L. Doob, Norman Levinson, and R. L. Wilder were appointed as representatives of the Society on the Board of Editors of the Annals of Mathematics for three years beginning January 1, 1949.

The report of the Committee on Aid to Devastated Libraries, as presented to the Council, was published in the January issue of this Bulletin.

The Council approved a report from the Policy Committee for Mathematics which recommended that a Union Conference be called for the discussion of the formation of an International Mathematical Union, this Conference to be held just before the 1950 International Congress and to be attended by representatives of all leading mathematical groups.

Certain recommendations of the Committee on the Role of the Society in Mathematical Publication were referred to the various editorial committees for consideration.

The Council voted to invite Professor Norbert Wiener to deliver the Josiah Willard Gibbs Lecture at the 1949 Annual Meeting in New York City.

Professor Marston Morse was re-elected as a representative of the Society on the Policy Committee for Mathematics for a period of four years beginning January 1, 1949.

Abstracts of the papers read follow. Presiding officers at the sessions for contributed papers were Professors E. J. McShane, W. L. Ayres, Richard Brauer, Hassler Whitney, J. S. Frame, Saunders MacLane, R. V. Churchill, Herbert Federer, S. C. Kleene and Dr. R. P. Boas.

Papers whose abstract numbers are followed by the letter "*t*" were read by title. Paper number 99 was presented by Professor Blumenthal, 101 by Professor Brauer, 116 by Professor Mann, 131 by Professor Seidel, 133 by Mr. Calabi, 135 by Professor Martin, 136 by Professor Goffman, 146 by Professor Green, 173 by Mr. Bott, 202 by Professor Zorn. Papers numbered 196, 198, 201, and 202 were presented at the joint session with the association for symbolic logic.

Algebra and Theory of Numbers

99. L. M. Blumenthal and D. O. Ellis: *Notes on metric lattices.*

This paper is concerned with lattice formulations of metric properties of the metric space $D(L)$ obtained from a normed lattice L by defining dist. $(a, b) = |a \cup b| - |a \cap b|$.

Lattice characterizations are given of metric betweenness and pseudo linear quadruples. It is shown that $D(L)$ is congruently contained in a line if and only if it contains no pseudo linear quadruples. If f is a bi-uniform mapping of one normed lattice onto another, equivalences are established among the properties: f preserves (1) order, (2) meets and joins, (3) distances, (4) norms. Properties (1) and (2) are (obviously) equivalent, while if L has a first or last element, any two of the properties (2), (3), (4) implies the third. (Received November 15, 1948.)

100*t*. Bailey Brown and N. H. McCoy: *The maximal regular ideal of a ring.*

An element a of the ring R is *regular* if there exists an element x of R such that $axa=a$, and an ideal is regular if and only if each element of the ideal is regular. It is shown that an arbitrary ring R contains a unique maximal regular two-sided ideal $M(R)$. Various properties of $M(R)$ are established, among them that (i) $M(R/M(R)) =0$; (ii) if $\mathfrak{b}$ is a two-sided ideal in R, $M(\mathfrak{b})=\mathfrak{b}\cap M(R)$; (iii) if R_n is the complete matrix ring of order n over R, then $M(R_n)=(M(R))_n$. The proofs are entirely elementary and hence, in particular, a simple proof is obtained that R_n is regular if and only if R is regular. This result has been proved by von Neumann for the case in which R has a unit element. If R satisfies the descending chain condition for right ideals, $R=M(R)\dotplus A$, where A is the annihilator of $M(R)$. This decomposition coincides with that of Marshall Hall (Trans. Amer. Math. Soc. vol. 48 (1940) pp. 391–404). (Received November 15, 1948.)

101. A. T. Brauer and Nathaniel Macon: *On the approximation of irrational numbers by the convergents of their continued fractions.*

Let ξ be any positive irrational number, A_n and B_n the numerator and the denominator of the nth convergent in the expansion of ξ as a regular continued fraction. We set $|\xi-A_n/B_n|=\lambda_n^{-1}B_n^{-2}$. Hurwitz (Math. Ann. vol. 39 (1891)) proved that there exist infinitely many $\lambda_n>5^{1/2}$ for every ξ, Vahlen (J. Reine Angew. Math. vol. 115 (1895)) that max $(\lambda_n, \lambda_{n+1})>2$, and Borel (J. Math. Pures Appl. (5) vol. 9 (1903)) that max $(\lambda_n, \lambda_{n+1}, \lambda_{n+2})>5^{1/2}$ for every n. It is proved in this paper that of each five consecutive λ_n either at least two exceed $5^{1/2}$ or at least one of them exceeds 3. Similar results are obtained for any set of $3m+2$ consecutive λ_n for $m=2, 3, \cdots$. Moreover estimates for the sums and products of consecutive λ_n are obtained. In particular, it is shown that lim inf $\{(\lambda_1+\lambda_2+\cdots+\lambda_m)/m\}>2.0169$. (Received November 15, 1948.)

102*t*. Leonard Carlitz: *Finite sums and interpolation formulas over* $GF[p^n, x]$.

Sums and interpolation formulas connected with polynomials in $GF[p^n, x]$ are discussed. Among the applications may be mentioned (1) criteria for the vanishing of certain sums; (2) the theorem that a polynomial $g(t)$ of degree less than p^{nm} is integral-valued if and only if $g(M)$ is integral for all M of degree less than m. (Received October 22, 1948.)

103*t*. Leonard Carlitz: *q-Bernoulli numbers and polynomials.*

"Numbers" η_m are defined by means of the symbolic formula $(q\eta+1)^m=\eta_m$, $m>1$; $\eta_0=1$, $\eta_1=0$; polynomials $\eta_m(x)$ in q^x by means of $\eta_m(x+1)-\eta_m(x)=mq^x[x]^{m-1}$, $\eta_m(0)=\eta_m$. Next define $\beta_m=\eta_m+(q-1)\eta_{m+1}$ and $q^x\beta_m(x)=\eta_m(x)+(q-1)\eta_{m+1}(x)$,

$\beta_m(0)=\beta_m$. Various properties of the η's and β's are derived. The main result is a partial generalization of the Staudt-Clausen theorem: $\beta_m=\sum N_{m,k}(q)/F_k(q)$, where $F_k(q)$ denotes the cyclotomic polynomial and $N_{m,k}(q)$ is a polynomial with integral coefficients. (Received October 22, 1948.)

104*t*. S. D. Chowla: *On a certain exponential sum.*

Let $\rho=e^{2\pi i/p}$ where p is a prime $\equiv 3 \pmod 4$. Suppose that the $(p-1)/2$ numbers $x_1, \cdots, x_{(p-1)/2}$, incongruent $(\mathrm{mod}\ p)$, have the property that (A) $\left|\sum_j^{(p-1)/2} \rho^{x_j}\right|^2 =(p+1)/4$. From the theory of the Gaussian sum it is evident that (A) is true if the x's are the numbers ar_m+b $(1\leqq m\leqq (p-1)/2)$ where r's are the quadratic residues of p. If $p=7$ or 11 the converse is also true. We show that the converse is not true when $p=31$ by the following example. For the set of x's take 0, 1, 2, 3, 5, 6, 8, 11, 12, 18, 19, 20, 23, 27, and 29. It is easy to verify that (A) is satisfied. But there exist no integers a and b such that this set is a rearrangement (mod 31) of the numbers ar_m+b where $r_1, \cdots, r_{15}$ are quadratic residues of 31. (Received November 16, 1948.)

105*t*. S. D. Chowla: *On a theorem of Hecke and Siegel.*

Without using the theory of binary quadratic forms the author proves the following result due to Hecke and Siegel (see Acta Arithmetica vol. 1 (1936) pp. 83–86). Let $\chi(n)$ denote a real primitive character $(\mathrm{mod}\ k)$; if there exists an $a>0$ such that $L(s)=\sum_1^\infty \chi(n)\cdot n^{-s}\neq 0$ for $s\geqq 1-a/\log k$ $(k>1)$, then $L(1)>b/\log k$ where $b=b(a)>0$. Details of the proof will appear with the paper. (Received November 15, 1948.)

106*t*. S. D. Chowla: *On difference sets.*

Veblen (Amer. Math. Monthly vol. 13 (1906) p. 46) raised the problem of finding 7 different integers $d_1, d_2, \cdots, d_7$ such that the 42 differences d_i-d_j $(i\neq j)$ are incongruent (mod 43). Singer (Trans. Amer. Math. Soc. vol. 43 (1938) pp. 377–385) proved that if m is equal to a power of a prime, we can find $(m+1)$ integers $d_1, \cdots, d_{m+1}$ such that the m^2+m differences d_i-d_j $(i\neq j)$ are incongruent $(\mathrm{mod}\ m^2+m+1)$. His proof is based on ideas of finite projective geometry. It seems very likely that the following assertion (A) is true: If m is *not* a power of a prime, it is *impossible* to find $(m+1)$ integers d_i $(i\leqq m+1)$ such that the m^2+m differences d_i-d_j $(i\neq j)$ are incongruent $(\mathrm{mod}\ m^2+m+1)$. A recent announcement of Bruck and Ryser (Bull. Amer. Math. Soc. vol. 54 (1948) pp. 649–650) implies the truth of (A) for infinitely many $m\equiv 1$ or 2 (mod 4). The author proves (A) for infinitely many $m\equiv 0$ or 3 (mod 4). As examples, the author proves (A) for $m=10$ (a case not covered by Bruck and Ryser) and when $m=159$. (Received November 15, 1948.)

107*t*. S. D. Chowla and K. G. Ramanathan: *On difference sets.*

Let $m\equiv 3$ (mod 4). The following problem is considered: What are the values of m for which there exist $(m-1)/2$ integers $d_1, d_2, \cdots, d_j$ $(j=(m-1)/2)$ such that the number of solutions of the congruence $d_i-d_j\equiv n \pmod m$ is independent of n? Such a set of d's is called a "difference set" $(\mathrm{mod}\ m)$. It is proved that: If $p\equiv 3$ (mod 4) is a prime and the set of numbers $d_1, d_2, \cdots, d_{(p-1)/2}$ is a difference set $(\mathrm{mod}\ p)$ and if there exists a k such that $d_i+d_j\equiv k \pmod p$ has no solution, then the d's are congruent $(\mathrm{mod}\ p)$ to the numbers $ar_1+b, ar_2+b, \cdots, ar_{(p-1)/2}+b$ in some order, where $r_1, \cdots, r_{(p-1)/2}$ are quadratic residues $(\mathrm{mod}\ p)$ and a and b are suitably chosen integers. (Received November 16, 1948.)

108*t*. S. D. Chowla and Paul Turán: *A congruence property of Ramanujan's τ-function.*

Ramanujan's function $\tau(n)$ is defined by $\sum_1^\infty \tau(n)x^n = x\prod_1^\infty (1-x^n)^{24}$ for $|x|<1$. Sharpening known results, the authors prove that: For almost all $n \leqq x$, we have $\tau(n) \equiv 0 \pmod{691^t}$ where $t > (1/690)(1-\epsilon) \log\log x$, where ϵ is an arbitrary positive number. (Received November 17, 1948.)

109*t*. F. Marion Clarke: *On the factorization of polynomials in n variables.* I.

For $f(x_1, \cdots, x_n)$, a finite, separable polynomial in n variables over a coefficient field K of characteristic zero, (1) a theorem of van der Waerden on transcendental extensions and a theorem of E. Noether on relations between the coefficients of a factorable polynomial are used to prove that any nontrivial factorization of f which is possible will occur over a finite algebraic extension of K; (2) the Riemann genus theorem and a property of dimension in the definition of a general point of an algebraic variety are used to show that if f is completely factored into absolutely irreducible factors over an algebraic extension field L of K: $f(x) = p_1(x) \cdots p_r(x)$, if $\mathfrak{p}_i$ and $\mathfrak{p}_j$ are prime ideals in $L[x_1, \cdots, x_n]$ generated by $p_i(x)$ and $p_j(x)$, $i \neq j$, and if $V_{\mathfrak{p}_i}$ and $V_{\mathfrak{p}_j}$ are the corresponding irreducible algebraic varieties $(i, j = 1, \cdots, r)$, then, in general, there exists no algebraic relation between the general points of $V_{\mathfrak{p}_i}$ and the general points of $V_{\mathfrak{p}_j}$. (Received November 16, 1948.)

110*t*. F. Marion Clarke: *On the factorization of polynomials in n variables.* II.

For a polynomial in n variables, $f(x_1, \cdots, x_n)$ over a coefficient field K of characteristic zero, a Σ_i-evaluation of f is defined and used to prove that when $n \geqq 3$ a necessary and sufficient condition for f to be irreducible over K is that there exist an infinite number of Σ_i-evaluations of f producing corresponding polynomials $\bar{f}(x_i, x_j)$ in the two variables x_i and x_j $(i \neq j)$, such that $\bar{f}$ is irreducible over K. It is shown that this property is stronger than is needed for the sufficiency condition, but that, according to a generalization of the Hilbert Irreducibility Theorem, it is minimal for the necessity condition. It is shown further that nonhomogeneous polynomials in two variables lend themselves to the methods of the theorem. (Received November 16, 1948.)

111*t*. Eckford Cohen: *An extension of Ramanujan's sum.*

This paper is concerned with the sum $c_q^s(n) = \sum_{(h,q^s)_s=1} e^{2n\pi hi/q^s}$ where q, s, n are positive integers and h ranges over non-negative integers $<q^s$ containing no sth power divisors in common with q^s other than 1. When $s=1$ this sum reduces to Ramanujan's sum $c_q(n)$ and several properties corresponding to those of $c_q(n)$ are proved. An analogue of $c_q^s(n)$ in the polynomial ring $GF[p^n, x]$ is discussed and an application is made to the problem of finding the number of representations of a polynomial as a sum of products. A corresponding asymptotic result for the rational case is mentioned. (Received November 9, 1948.)

112*t*. Harvey Cohn: *On a disproof of one of Minkowski's conjectures.*

Minkowski's conjecture concerns the minimum constant c for which every lattice of unit density has a nontrivial point satisfying the inequality $|x|^p+|y|^p\leqq c$. The shape of the critical lattice is determined by the minimum parallelogram with one vertex at the origin and remaining vertices on the curve $|x|^p+|y|^p=1$. The conjecture was that the critical lattice would belong to the parallelogram with horizontal base if $1<p<2$ and to the one with a base at 45° if $p>2$. By using asymptotic expansions the author tests these two symmetrical parallelograms for relative maxima and minima, finding, for instance, that the symmetrical parallelograms do not always provide the relative maxima and minima. In so doing, the author verifies C. S. Davis' disproof of Minkowski's conjecture, and reveals complications in the behavior of the inscribed parallelogram viewed as a function of the slope of a side. (Received December 8, 1948.)

113. Melvin Dresher: *Continuous games with polynomial pay-off functions.*

Consider a two-person zero-sum game, in which each player chooses a real number between zero and one. Let $M(x, y)$ be the pay-off to the first player if he chooses x and the second player chooses y and let $F(x)$ and $G(y)$ be the cumulative distribution functions of the frequencies with which the players choose these numbers. Then $F^*(x)$, $G^*(y)$ is said to be a *solution* of the game if $\max_F \int_0^1\int_0^1 M(x, y)dF(x)dG^*(y) = \min_G \int_0^1\int_0^1 M(x, y)dF^*(x)dG(y) = \int_0^1\int_0^1 M(x, y)dF^*(x)dG^*(y)$. If $M(x, y) = \sum_{j=0}^n \sum_{i=0}^m a_{ij}x^iy^j$, and if neither $\int_0^1 M(x, y)dG^*(y)$ nor $\int_0^1 M(x, y)dF^*(x)$ is identically constant, then every solution $F^*(x)$, $G^*(y)$ is a pair of step-functions having at most $[m/2]+1$ and $[n/2]+1$ jumps respectively. The jumps in $F^*(x)$ occur at the maximum points of $\int_0^1 M(x, y)dG^*(y)$. If both $\int_0^1 M(x, y)dG^*(y)$ and $\int_0^1 M(x, y)dF^*(x)$ are constant then the solution of the game is in terms of m moments of $F^*(x)$ and n moments of $G^*(y)$. If $\int_0^1 M(x, y)dG^*(y)$ is constant and $\int_0^1 M(x, y)dF^*(x)$ is not, then the solution consists of $[m/2]+1$ jumps in $F^*(x)$ and n moments of $G^*(y)$. These results provide a method of computing all solutions of games with polynomial pay-off functions. (Received November 15, 1948.)

114. N. J. Fine: *Basic hypergeometric series and arithmetic applications.* Preliminary report.

Let $F(\alpha, \beta; t:x) = \sum_{n=0}^\infty ([n; \alpha]/[n; \beta])t^n$, where $[0; z]=1$, $[n; z]=(1-xz)(1-x^2z)\cdots(1-x^nz)$ for $n>0$. By comparatively elementary methods it is shown that each of $F(\alpha x, \beta; t:x)$, $F(\alpha, \beta x; t:x)$, $F(\alpha, \beta; tx:x)$ can be expressed in terms of $F(\alpha, \beta; t:x)$. These functional equations lead to other transformations of F, such as $(1-t)F(\alpha, \beta; t:x) = (1-t')f(\alpha', \beta'; t':x)$, where $\alpha'=\alpha t/\beta$, $\beta'=t$, $t'=\beta$. Other transformations lead, on suitable specialization of the parameters, to classical identities of Euler, Jacobi, Gauss, and to results on the "false" ϑ-functions of Rogers, the "mock" ϑ-functions of Ramanujan, and the classical ϑ-functions. A number of these results are believed to be new, and some have simple arithmetic interpretations. For example, define the *rank* of a partition as the excess of the largest part over the number of parts. Let $D_r(n)$ be the number of partitions of n into *distinct* parts, the rank of each partition being r; let $U_{2r+1}(n)$ be the number of partitions of n into *odd* parts, the largest being $2r+1$. Then (*) $U_{2r+1}(n)=D_{2r+1}(n)+D_{2r}(n)$. Euler's well known theorem follows by summing (*) for all $r\geqq 0$. Also, let $L(n)$ be the number of partitions of n into distinct parts the least of which is odd. Then $L(n)$ is odd if and only if n is a square. (Received November 16, 1948.)

115. R. H. Fox: *Free differential calculus and the Jacobian matrices of a group.*

Consider the integral group ring P of a free group F and the endomorphism $u \to u^0$ of P which maps F into the identity 1. To each generator x_j of F there is a linear mapping $u \to \partial u/\partial x_j$ of P into itself which is determined uniquely by the conditions (1) $\partial(uv)/\partial x_j = (\partial u/\partial x_j) \cdot v^0 + u \cdot (\partial v/\partial x_j)$, (2) $\partial x_k/\partial x_j = \delta_{jk}$. The formula $u = u^0 + \sum_j (\partial u/\partial x_j) \cdot (x_j - 1)$ holds, so that u is determined up to an additive constant by its partial derivatives $\partial u/\partial x_j$. Let G be a group determined by a set $x_1, x_2, \cdots$ of generators and a set $u_1 = 1, u_2 = 1, \cdots$ of defining relations and denote by ϕ the natural homomorphism of F upon G (and also the induced homomorphism of P upon the integral group ring of G). The matrix $\|(\partial u_i/\partial x_j)^\phi\|$ is called a *Jacobian* of the group G. The Jacobian matrices of G all belong to a single equivalence class. From this group invariant the various previously known invariants may be derived by specialization. The differentiation concept may be generalized in several ways and there are various applications of this calculus, particularly to free groups. (Received November 18, 1948.)

116. Marshall Hall and H. B. Mann: *On a canonical form of the Euclidean algorithm.*

The Euclidean algorithm is said to be valid in a commutative ring R if there exists on R a function $E(a)$ with the following properties. (1) $E(a)$ is an element of a well ordered set M. (2) $E(a)$ is the first element of M if and only if $a = 0$. (3) To every a, b in R, $b \neq 0$, there exist elements $g, r \subset R$ such that $a = bg + r$, $E(0) \leqq E(r) < E(b)$. The algorithmic chain is a chain of subsets S_a of elements of R defined as follows. S_0 consists of the 0 of R. S_a consists of all $x \subset R$ such that every residue class mod x contains at least one element in S_b with $b < a$. The following theorem is proved. The Euclidean algorithm is valid in R if and only if the algorithmic chain contains all elements of R. The function $A(x) = a$ if $x \subset S_a$, $x \not\subset S_b$ with $b < a$, is called the canonical algorithm if it is defined for every x. The function $A(x)$ satisfies the inequality $A(xy) \geqq A(x)$ if $xy \neq 0$, $A(xy) > A(x)$ if $xy \neq 0$, $A(y) > 1$. Every element in S_a decomposes into at most $(a-1)$ prime factors. (Received November 10, 1948.)

117*t*. Frank Harary: *On atomic Boolean-like rings.*

A zero ring is a ring of characteristic two in which the product of any two elements is zero. Any two zero rings of the same cardinal power are isomorphic. Every zero ring N has a base B such that each element of N can be uniquely written as the sum of a finite number of elements of B. For brevity, let "BLR" denote "Boolean-like ring." An atomic BLR is a BLR whose Boolean subring is atomic. A nilpotent element η of an atomic BLR, H, is called nil if $b\eta = 0$, where b is any atom of the Boolean subring J of H. Similarly η is called simple if for some atom b of J, $b\eta = \eta$. Let H be an atomic BLR; N its nilpotent ideal; B a base for N, then H is called simple if each element of B is either nil or simple. Our principal theorem is that every atomic BLR is isomorphic to a simple one. The proof is by transfinite induction. (Received November 17, 1948.)

118*t*. Frank Harary: *On n-dimensional projective algebras.* Preliminary report.

n-dimensional projective algebras (PA's) are defined by a suitable modification of the Everett-Ulam postulates. The elementary theorems on n-dimensional PA's analogous to those for ordinary (that is, 2-dimensional) PA's follow at once. It is shown that an n-dimensional PA can be synthesized from any non-atomless Boolean algebra B. The isomorphism types of finite n-dimensional PA's are completely determined. (Received November 17, 1948.)

119. Frank Harary: *The structure of finite projective algebras.* Preliminary report.

The Everett-Ulam postulates for projective algebra (PA) immediately imply a duality theorem relating x and y projections. It is shown that a PA can be synthesized from any nonatomless Boolean algebra B, with a fixed atom p, by defining (1) $0_x = 0_y = 0$, (2) $a_x = p$ and $a_y = a$, when $a \in B$, $a \neq 0$, (3) $p \square a = a$ for all $a \in B$, (4) $a \square b = 0$ if a or b is 0. The isomorphism types of finite PA's are completely determined. The number of isomorphism types of PA's of 2^m elements is equal to the number of distinct factorizations of m into 2 factors. (Received November 17, 1948.)

120*t*. G. B. Huff: *Tentative methods in diophantine equations.*

If a diophantine equation is equivalent to one of the form $w_1w_2 = f(w_3, w_4, \cdots, w_n)$, where f is a polynomial with integer coefficients, it is said to be solvable by factoring. It is shown that this idea provides a complete theory for the system $z_0^2 - z_1^2 - z_2^2 - \cdots - z_m^2 = b$, $z_0 - z_1 - z_2 - \cdots - z_m = a$. Applications are given, particularly to problems in algebraic geometry. (Received November 15, 1948.)

121. L. C. Hutchinson: *Notes on the number of leaves of an alternating tensor.*

It has long been known that the smallest number $m(r)$ of leaves (simple tensors) in terms of which a bivector, or alternating tensor of valence 2 (2 indices) and rank r can be expressed is $r/2$ (r is always even in this case). For alternating tensors of higher valence the classification is more complicated, and $m(r)$ becomes a multiple-valued function of r. Some bounds for $m(r)$ are given, with particular attention to the trivector (valence 3). The results for the trivector differ from those given by R. Weitzenböck (K. Akademie van Wetenschappen, Proceedings vol. 32 (1929) pp. 248–250, vol. 40 (1937) pp. 312–315; Monatshefte für Mathematik und Physik vol. 48 (1939) pp. 129–140). The best lower and upper bounds so far determined (in the real) are $r/3$ and $(r-5)(r+3)/4$ ($r>7$), respectively. (Up to $r=8$ the exact values are known.) If complex transformations are allowed, this result can be strengthened. (Received November 17, 1948.)

122. B. W. Jones: *On m-universal quadratic forms.*

If A is the nonsingular matrix of an n-ary form f in a field F and if for every m-rowed nonsingular symmetric matrix B in F there is a matrix T in F and of rank m such that $T^TAT = B$, we call f an "m-universal form." The form f is shown to be m-universal in the field $F(p)$ of p-adic numbers if $n > m+2$ and conditions are found for the cases $n = m+2$ and $n = m+1$. From these conditions follow, by Hasse's results, criteria that a form be m-universal in the field of rational numbers. In the field of real numbers the form f is m-universal if and only if there is a matrix T of rank m such that T^TAT is the zero matrix of order m. (Received November 13, 1948.)

123t. Irving Kaplansky: *Elementary divisors and modules.*

Conditions are found on a ring R in order that matrices over R (both finite and infinite) admit reduction to diagonal form. This is applied to the decomposition of an R-module into a direct sum of cyclic modules. The uniqueness of the decomposition is studied. (Received October 29, 1948.)

124. J. C. C. McKinsey: *Isomorphism of games and strategic equivalence.*

Two zero-sum n-person games, v and v', are called *S-equivalent* if there exists a positive number k, and n numbers $a_1, \cdots, a_n$, whose sum is zero, such that, for every subset T of the set of players, $v'(T)=k\cdot v(T)+\sum_{i\in T}a_i$. Games are called *isomorphic* if there exists a one-to-one correspondence between the sets of imputations which, for every subset T of the set of players, preserves dominance with respect to T. It is shown that games are S-equivalent if and only if they are isomorphic. The notion of strategic equivalence is an intuitive one, but it is sufficiently sharp that intuitive arguments can be presented to show that isomorphism is a necessary condition for strategic equivalence, and that S-equivalence is a sufficient condition. From the result mentioned above it then follows that games are strategically equivalent (in the intuitive sense) if and only if they are S-equivalent. (Received November 15, 1948.)

125. Gordon Pall: *Sums of two squares in a quadratic field.*

It is shown that the number of representations of a binary quadratic form as a sum $(a_1x+b_1y)^2+(a_2x+b_2y)^2$ of squares of two linear forms with integral coefficients is (when the obviously necessary conditions of having a square determinant, integral coefficients, and being non-negative are satisfied) equal to the number of representations of the divisor of the form as the sum of two squares of integers, or equal to double this number, according as the determinant is or is not zero. As an application, it is shown that simple formulae can be obtained for the number of representations of an integer in a quadratic field as the sum of squares of two integers in that field. The number of such representations is finite for all real quadratic fields, but infinite (or zero) for all imaginary quadratic fields, with the single exception of the ring of Gauss integers $a+bi$ ($i^2=-1$) for which the number of representations of a nonzero number is finite. As an example, the number of integral solutions of $a+2ki=(a_1+b_1i)^2+(a_2+b_2i)^2$ is given by $4\epsilon_\alpha\prod(1+\beta_i)\prod(1+\gamma_j)(1+\gamma_j+\delta_j)$, where $a^2+4k^2=2^\alpha q_1^{2\beta_1}\cdots q_r^{2\beta_r}p_1^{2\gamma_1+\delta_1}\cdots p_s^{2\gamma_s+\delta_s}$; here $\alpha\geqq 0$; $q_1, \cdots, q_r$ denote distinct primes of the form $4n+3$, $r\geqq 0$; $p_1, \cdots, p_s$ denote distinct primes of the form $4n+1$, $s\geqq 0$, and $p_j^{\gamma_j}$ is the power of p_j dividing the g.c.d. (a, k); and $\epsilon_0=1$, $\epsilon_\alpha=|\alpha-3|$ if $\alpha\geqq 2$. (Received December 1, 1948.)

126t. Hugo Ribeiro: *A remark on "uniform" lattices.*

A finite modular lattice is called u-uniform if and only if each convex complemented sublattice of dimension 2 has $u+3$ elements. In an author's paper to appear in Comment. Math. Helv. the following facts are proved: Disregarding non-Desarguesian projective geometries, Boolean algebras and lattices of all subgroups of elementary prime-power abelian groups are the only complemented u-uniform lattices for which u is an indecomposable integer (1 in the first case, a prime in the second case). p-uniformity (p prime) together with other simple conditions such as the cyclic character of all join-irreducible elements characterize the lattice of all subgroups of the primary p-component of a finite abelian group. It is now a consequence of the above facts that: Each p-uniform lattice for which the following conditions are satisfied is an auto-dual lattice: (1) if $x \neq x \cup y \neq y$ then for each z there are u and v with $u \neq u \cup v = x \cup y \cup z \neq v$ (join-irreducible elements are cycles), (2) the number of elements following an element immediately is at most the number of atoms, (3) all maximal cycles are of the same dimension. (Received November 19, 1948.)

127t. C. E. Rickart: *The uniqueness of norm problem in Banach algebras.*

A B-algebra is said to possess a unique norm provided any two norms under which it is a B-algebra are necessarily equivalent. It is known that the norm is unique in a semi-simple, commutative B-algebra (Gelfand, Rec. Math. (Mat. Sbornik) N.S. vol. 9 (1941) pp. 1–23) and in the B-algebra of all bounded operators on a B-space (Eidelheit, Studia Mathematica vol. 9 (1940) pp. 97–105). It is shown here that the same is true for any B-algebra which possesses a family of minimal left ideals such that $xL=(0)$ for every L in the family implies $x=0$. This includes the case of a primitive B-algebra with minimal ideals and hence the Eidelheit result. The norm is also unique in any B-algebra with an adjoint operation x^* which is either essential in the Raikov sense (C. R. (Doklady) Acad. Sci. URSS N.S. vol. 54 (1946) pp. 387–390) or satisfies a condition $\|x\|^2 \leqq k\|x^*x\|$. This includes the case of a B-algebra of bounded operators on Hilbert space where x^* is the operator adjoint but the norm need not be the bound. (Received November 10, 1948.)

128t. C. E. Rickart: *Homomorphisms of one Banach algebra into a second.*

Let $h(x)$ denote an arbitrary homomorphism of one B-algebra B *into* a second B-algebra A. It is shown that $h(x)$ is automatically continuous in each of the following cases: (1) $h(x)$ is *onto* and A is a semi-simple B-algebra whose norm is uniquely determined up to equivalence, (2) A is semi-simple and commutative, (3) A possesses an adjoint operation x^*, which is either essential in the sense of Raikov (C. R. (Doklady) Acad. Sci. URSS N.S. vol. 54 (1946) pp. 387–390) or satisfies the condition $\|x\|^2 \leqq k\|x^*x\|$, provided $h(B)^*=h(B)$. (Received November 10, 1948.)

Analysis

129. Nachman Aronszajn: *Hilbert spaces and conformal mappings.*

A formula for the projection on the sum of two closed linear subspaces of a Hilbert space in terms of projections on the subspaces is applied to the computation of conformal mappings of simply and multiply connected domains. The link between the Hilbert space theory and the conformal mapping is found in the theory of reproducing kernels, especially Bergman's kernels which are the reproducing kernels of classes of analytic functions (of one or more complex variables) regular and square integrable in a domain. (Received November 18, 1948.)

130t. Frederick Bagemihl: *A theorem on infinite products of transfinite cardinal numbers.*

Let α be a transfinite limit-number, and let its normal form be $\sum_{1 \leqq k \leqq n} \omega^{\delta_k}=\alpha$, where n is a natural number and $\delta_1 \geqq \delta_2 \geqq \cdots \geqq \delta_n > 0$. Let $\{\sigma_\xi\}_{\xi<\alpha}$ be an increasing sequence of ordinals such that $\lim_{\xi<\alpha}\sigma_\xi=\lambda$, and subject, if $n>1$, to the following condition: if $\beta=\sum_{1\leqq k\leqq n-1}\omega^{\delta_k}$, and $\lim_{\xi<\beta}\sigma_\xi=\eta$, then $-\overline{\eta+\lambda}\leqq\overline{\omega^{\delta_n}}$. Then $\prod_{\xi<\alpha}\aleph_{\sigma_\xi}=\aleph_\lambda^{\bar{\alpha}}$. In particular, if α is a transfinite limit-number, and μ is an arbitrary ordinal number, then $\prod_{\xi<\alpha}\aleph_{\mu+\xi}=\aleph_{\mu+\alpha}^{\bar{\alpha}}$. The method of proof is essentially one due to Tarski (Fund. Math. vol. 7 (1925) pp. 1–14). (Received November 16, 1948.)

131. E. F. Beckenbach, Wladimir Seidel, and Otto Szasz: *Recurrent determinants of orthogonal polynomials.*

A study is made of recurrent determinants $D_n=[c_{\nu+\mu}]$ and $E_n=[c_{\nu+\mu+1}]$, ν, $\mu=0, 1, 2, \cdots, n-1$; n a positive integer, where $c_m=Q_m(x)$ is a polynomial of degree m of one of the following classes: Legendre, Hermite, Laguerre, and ultraspherical, suitably normalized. Explicit formulas for the determinants are obtained in a variety of cases. It is shown, for instance, that for Legendre polynomials $D_n\equiv[P_{\nu+\mu}(x)]=2^{-(n-1)^2}(x^2-1)^{(n-1)n/2}$ and $E_n\equiv[P_{\nu+\mu+1}(x)]=[(x^2-1)/4]^{(n-1)n/2}\cdot[(x+1)^n+(x-1)^n]/2$. Furthermore, all minors of D_n are positive for $x>1$. This is applied to the Stieltjes and Hamburger moment problems. (Received November 15, 1948.)

132. H. D. Block: *Explicit solution of certain singular integral equations.*

The equation $f(x)=\lambda\int_a^\infty K(|x-y|)g(y)dy$ is solved by a simple method for the unknown function $g(y)$ in certain special cases. $K(|x-y|)$ as a function of x is assumed to satisfy the same linear homogeneous differential equation in both the regions $a\leqq x\leqq y$; $y\leqq x$. The method of solution consists in constructing a linear differential equation with constant coefficient which $g(y)$ must satisfy. Necessary and sufficient conditions are then found for the cases when $K(|x-y|)=e^{-k|x-y|}$; and $K(|x-y|)=|x-y|e^{-k|x-y|}$; for $-\infty\leqq a<\infty$. The solution of the equation of the second kind $f(x)=g(x)-\lambda\int_a^\infty K(|x-y|)\,g(y)dy$ is also included for these cases. For the homogeneous equation, $g(x)=\lambda\int_a^\infty K(|x-y|)g(y)dy$, the complete solutions are found, and the characteristic values of λ are shown to form a continuous spectrum. (Received November 9, 1948.)

133. E. Calabi and Aryeh Dvoretzky: *Convergence and sum factors for series of complex numbers.*

Let Z be a given bounded set, and denote by $\tilde{Z}$ the closure of the convex set spanned by Z. Theorem 1: $0\in\tilde{Z}$ is the necessary and sufficient condition that given any null sequence $(a_n)_{n=1}^\infty$ of complex numbers, there exist $\zeta_n\in Z$ so that (I) $\sum_{n=1}^\infty\zeta_na_n$ converges. Theorem 2: If, in addition, $\sum_{n=1}^\infty|a_n|=\infty$ the necessary and sufficient condition that $\zeta_n\in Z$ may be chosen so that (I) converges to any preassigned limit is that 0 be an interior point of $\tilde{Z}$. In this case one can also choose $\zeta_n\in Z$ so that the derived set of $\sum_{n=1}^N\zeta_na_n$ is any preassigned closed, connected set. The case $Z=\{1, -1\}$ of Theorem 1 was proved by A. Dvoretzky and C. Hanani (C. R. Acad. Sci. Paris vol. 225 (1947) p. 516), while in the special case where Z consists of the kth ($k\geqq 3$) roots of unity Theorem 2 was established by H. Hornich (Monatshefte für Mathematik und Physik vol. 45 (1937–1938) p. 432, vol. 46 (1938–1939) p. 266). Generalizations to the case when ζ_n is to be chosen from a varying Z_n are also given. Applications to sequences of functions are included. The methods and results carry over to sequences of vectors in E^r and more general spaces. (Received November 13, 1948.)

134*t*. J. W. Calkin: *Spaces with inner products defined in terms of generalized limits.*

If $\mathfrak{H}$ is a Hilbert space, La_n a generalized (Banach) limit defined for all bounded sequences $\{a_n\}$ of complex numbers, we denote by $\mathfrak{H}_L$ the space of all sequences weakly convergent to zero with $(\{f_n\}, \{g_n\}) = L(f_n, g_n)$, sequences differing by a strongly convergent sequence being regarded as equal. This possibly incomplete Euclidean space, whose dimension number is c, was used in a previous paper (Ann. of Math. vol. 42 (1941) pp. 839–873) to study the ring of bounded operators on $\mathfrak{H}$ reduced modulo the totally continuous ones. If S denotes the Stone-Čech compactification of the integers I and $a(p)$ is the continuous function which $\{a_n\}$ induces on S, then for $p \in S - I$, $a(p) = La_n$ defines a generalized limit. If L is any generalized limit, then $La_n = \int_s a(p) d\mu$ where μ is a uniquely determined regular countably additive measure defined on Borel sets in S with $\mu(S) = 1$, $\mu(I) = 0$ (Nakamura and Kakutani, Proc. Imp. Acad. Tokyo vol. 19 (1943) pp. 224–229). It is shown here the $\mathfrak{H}_L$ is complete if μ assumes only a finite number of values, but not otherwise. (Received October 21, 1948.)

135. R. H. Cameron and W. T. Martin: *Nonlinear integral equations.*

This paper deals with the integral equation (1) $y(t) = x(t) + \int_0^t F(t, s, x(s)) ds$ on the interval I: $0 \leq t \leq 1$ and the corresponding equation (2) with fixed upper limit unity. Both of these equations are considered as special cases of the functional equation (3) $y(t) = x(t) + \Lambda(x \mid t)$, where the functional Λ satisfies certain smoothness conditions and (3) is a 1–1 transformation of the space C of continuous functions vanishing at the origin into itself. It is shown that under suitable restrictions on F or Λ the equations (1), (2), (3) have a solution of the form $x(t) = \text{L. I. M.}_{n \to \infty} \sum \Psi_m(y) A_m(t)$ where m represents an n-fold index $m_1, \cdots, m_n$, each component of which goes from 0 to n in the sum. If certain additional conditions are satisfied by F or Λ the expression for $x(t)$ can be replaced by a modified sum which is pointwise summable in the appropriate infinite-dimensional Abel sense. Thus absolute results as well as statistical results are obtained. (Part of the work done on this paper by one of the authors (R. H. C.) was carried on at the Institute for Numerical Analysis with the financial support of the Office of Naval Research of the Navy Department.) (Received October 25, 1948.)

136. L. W. Cohen and Casper Goffman: *Note on non-archimedean metrization.*

A non-archimedean metric is a function $\delta(x, y)$ on pairs of a space S to a Hahn field $\mathcal{J}$ such that (1) $\delta(x, y) \geq 0$, $\delta(x, y) = 0$ if and only if $x = y$; (2) $\delta(x, y) = \delta(y, x)$; (3) $\delta(x, z) \leq \delta(x, y) + \delta(y, z)$. Conditions are given under which a space S is homeomorphic to a non-archimedean metric space. These include conditions of the sort given previously by the authors (*A theory of transfinite convergence*, to appear in Trans. Amer. Math. Soc.) and the additional restriction that neighborhoods be closed sets. It follows that certain topological groups which do not satisfy the first denumerability axiom of Hausdorff may be metrized in the sense considered here. (Received November 18, 1948.)

137. V. F. Cowling: *Summability and analytic continuation.*

A new set of matrix methods of summability is introduced. The elements of the matrix (a_{nm}) are defined, for each real number r satisfying the inequality $0 \leq r < 1$, as $a_{nm} = 0$, $m < n$; $a_{nm} = (1 - r)^{n+1}(C_{m,n}) r^{m-n}$, $m \geq n$. It is shown that each matrix of this set determines a regular summability method. A sequence or series which is summable by one of these matrices will be said to be summable F_r (the real number r depending on the matrix employed). The question of consistency of the various methods F_r among themselves is discussed. A general theorem is obtained as to the regions in which these methods will sum a power series. We illustrate this theorem in the case of the series $\sum z^n$. The series $\sum z^n$ is summable F_r for each real number r satisfying the inequality $0 \leqq r < 1/2$ to sum $1/(1-z)$ in any closed region contained in the region common to $|z+r/(1-2r)| < (1-r)/(1-2r)$ and $|z| < 1/r$. It is summable $F_{1/2}$ to sum $1/(1-z)$ in any closed region contained in the region common to $|z| < 2$ and $R(z) < 1$. Finally it is summable F_r for each real number r satisfying the inequality $1/2 < r < 1$ to sum $1/(1-z)$ in any closed region contained in the region common to $|z+r/(1-2r)| > (1-r)/(2r-1)$ and $|z| < 1/r$. (Received November 12, 1948.)

138*t*. R. J. Duffin: *Representation of Fourier integrals as sums*. II.

Let the function $\phi(x)$ have a derivative which has bounded variation in the interval $(1, \infty)$ and which has the limit zero at infinity. Let $\Phi(x) = \phi(1/x)/x$ have the same property. Under these hypotheses the following conclusions are drawn: For positive x a continuous function $g(x)$ is defined by the series $g(k/x)/x = \phi(x) - \phi(3x) + \phi(5x) - \cdots$. Here $k = (\pi/2)^{1/2}$. Likewise, a continuous function $G(x)$ is defined by the series $G(k/x)/x = \Phi(x) - \Phi(3x) + \Phi(5x) - \cdots$. These series are summable by the Cesaro mean. The functions $g(x)$ and $G(x)$ are a pair of Fourier sine transforms; that is, $kG(x) = \int_0^\infty \sin xtg(t)dt$ for positive x. The integral is summable by the Cesaro mean. Any function $\phi(z)$ which is analytic and uniformly bounded in the sector $-\epsilon < \arg z < \epsilon$ satisfies the conditions of this theorem. (Received November 16, 1948.)

139*t*. R. J. Duffin and A. C. Schaeffer: *Functions whose Fourier Stieltjes coefficients approach zero*.

Let $f(z) = \int_{-\pi}^{\pi} e^{izt} d\rho(t)$, where $\rho(t)$ is a function of bounded variation. Let k_n, $n = 1, 2, \cdots$, be a bounded sequence of real numbers, and suppose $f(n+ik_n) \to 0$ as $n \to \infty$. It is shown in this note that a necessary and sufficient condition that $f(x) \to 0$ as $x \to +\infty$ is that $\rho(t)$ be continuous at $t = \pi$. This is closely related to certain results of Rajchman (Math. Ann. vol. 101 (1929) pp. 686–700), and Verblunsky (Proc. London Math. Soc. vol. 34 (1932) pp. 526–560). Their theorems concern "n" rather than "$n+ik_n$." The theorem is proved by a simple application of a complex variable technique developed previously by the writers (Amer. J. Math. vol. 67 (1945) pp. 141–154). (Received November 18, 1948.)

140*t*. M. M. E. Eichler: *On the differential equation* $u_{xx} + u_{yy} + N(x)u = 0$.

This paper is closely connected with S. Bergman's recent work on differential equations. General solutions can be written as series $u = \sum_0^\infty p_\nu(x) f_\nu(z)$ where $f_\nu(z)$ are analytic functions of $z = x+iy$. Two such series developments are discussed: the "ascending series," where $f_\nu(z) = df_{\nu+1}(z)/dz$, and the "descending series," where $f_{\nu+1}(z) = df_\nu(z)/dz$. The ascending series can be written as an integral operator $u = f_0(z) - \int_{z_0}^{z} G(x, z-\zeta) f_0(\zeta) d\zeta$ working on $f_0(z)$. Specially distinguished operators of this type give insight into the analytic behavior of series $\sum q_\lambda(x) \exp(i\lambda y)$ of particular solutions, obtained by the method of separation of variables, outside of their domain of convergence. Examples of differential equations, important in the applications, are given where explicit expressions for the "generating solutions" G are known. Two methods to calculate G explicitly in other cases are explained. The convergence of the ascending and descending series is investigated, particularly in the neighborhood of a singular point of N where N is of the form x^{-2} times a power series in x^{ρ} ($\rho>0$). A connection between both series is briefly touched in a special case. At last a class of differential equations in an arbitrary number of variables is mentioned to which methods and results of this paper can largely be extended. (Received October 22, 1948.)

141. H. Margaret Elliott: *On line and surface integral approximations to harmonic and analytic functions satisfying an integrated continuity condition.*

Results on approximation to harmonic and analytic functions are established when both the degree of approximation and the continuity properties of the function are measured by (1) a line (cf. E. S. Quade, Duke Math. J. vol. 3, pp. 529–543 for the trigonometric case) and (2) a surface integral. Let C be an analytic Jordan curve. Let $f(s)\in L^p$, $p\geqq 1$, s arc-length measured along C. The integral modulus of continuity $\omega_L^p(\delta)$ of $f(s)$ on C is defined as $\max_{0\leqq h\leqq\delta}[\int_0^{\sigma}|f(s+h)-f(s)|^p ds]^{1/p}$, where σ is the length of C. It is proved, for example, that if $f(z)$ is analytic interior to C and if $f^{(k)}(z)\in L^p$, $p\geqq 1$, on C and has integral modulus of continuity $\omega_L^p(\delta)$ on C, then for each n, $n=1, 2, \cdots$, there exists a polynomial $\pi_n(z)$ in z of degree n such that $[\int_C|f(z)-\pi_n(z)|^p ds]^{1/p}\leqq M\omega_L^p(1/n)/n^k$. Conversely if there exist polynomials $\pi_{n_j}(z)$ of respective degree n_j, $j=1, 2, \cdots$, such that $[\int_C|f(z)-\pi_{n_j}(z)|^p ds]^{1/p}\leqq\Omega(n_j)/n_j^k$, $p\geqq 1$, $k\geqq 1$, it is shown under suitable restrictions on $\Omega(x)$ and the sequence n_j that $f(z)$ is equal almost everywhere on C to a function $f_1(z)$ which is analytic interior to C; furthermore $f_1^{(k)}(z)$ has integral modulus of continuity on $C\,\omega_L^p(\delta)\leqq L[\delta\int_{n_1}^{1/\delta}\Omega(x)dx+\int_{n/\delta}^{\infty}(\Omega(x)/x)dx]$, $0<\delta\leqq 1/n_1$. Analogous results are obtained for the surface integral. (Received November 15, 1948.)

142*t*. Evelyn Frank: *Convergence of C-fractions*. Preliminary report.

Parabolic and other convergence regions of C-fractions $1+a_1z^{\alpha_1}/1+a_2z^{\alpha_2}/1+\cdots$ are obtained. Value regions for these continued fractions are also shown. (Received October 21, 1948.)

143. W. H. J. Fuchs: *On the growth of functions of mean type.*

$\{\lambda_n\}$ is a sequence of positive numbers, $\lambda_{n+1}-\lambda_n>c>0$. N. Levinson and others have discussed under which conditions on $\{\lambda_n\}$, $\lim\sup_{n\to\infty}\lambda_n^{-1}\log|f(\lambda_n)|=\lim\sup_{x\to\infty}x^{-1}\log|f(x)|$ (1) for every function $f(z)$ regular in $x\geqq 0$ and satisfying $|f(z)|=O(e^{\pi L|z|})$ in this half-plane. A refinement of Levinson's method shows that the following conditions are necessary and sufficient to ensure (1). (i) $\phi(r)=\sum_{\lambda_n<r}\lambda_n^{-1}-L\log r\to\infty$. (ii) For $Y>X\geqq N(\epsilon)$, $\phi(Y)-\phi(X)>-\epsilon$. (Received November 11, 1948.)

144. A. M. Gleason: *Subalgebras of a measure algebra.*

A measure algebra is a σ-Boolean algebra on which there can be defined a finite, non-negative, completely additive measure function μ, such that $\mu(a)=0$ only if a is the null element of M. Maharam (Proc. Nat. Acad. Sci. U. S. A. vol. 28 (1942) pp. 108–111) has characterized the structure of measure algebras. Using a similar technique, the author characterizes the structure of pairs consisting of a measure algebra M and a subalgebra L. As an example of such a pair consider the algebra M of all measurable sets, modulo null sets, of the direct product of two measure spaces X and Y, and let L consist of those sets of the form $E\times Y$ where E is a measurable subset of X. A more recondite example can be derived from the preceding by considering a principal ideal aM of M as a measure algebra and the set aL as a subalgebra. It is shown that the latter example is, indeed, the most general. (Received November 12, 1948.)

145. A. W. Goodman: *The Schwarz-Christoffel transformation and p-valent functions.* Preliminary report.

Subclasses $S(p)$ and $C(p)$ of the class of functions p-valent in the unit circle are introduced which are generalizations of the classes of univalent functions which map the unit circle on starlike and convex regions respectively. By using a general form of the Schwarz-Christoffel transformation which maps the unit circle onto a p-sheeted domain bounded by straight line segments, it is possible to prove sharp theorems for functions of class $S(p)$ or $C(p)$. In particular bounds are obtained for the coefficients and for $|f(z)|$. If $f(z)=a_1z+a_2z^2+a_3z^3+\cdots$ is in $S(2)$ and has all coefficients real, then $|a_3|\leqq 5|a_1|+4|a_2|$. This is a special case of an earlier conjecture. As a by-product of the methods two more proofs that $S(2)=\pi^2/6$ are given. (Received November 9, 1948.)

146. J. W. Green and William Gustin: *Quasiconvex sets.*

Let I be the closed unit real number interval. Any subset Δ of I containing at least one number interior to I is called a quasiconvexity generating set. A set Q in a finite-dimensional real vector space is Δ convex if for every real number α of Δ and every two points a and b lying in Q the point $\alpha a+\beta b$ $(\alpha+\beta=1)$ also lies in Q. The graph of a solution of the functional equation $\phi(x+y)=\phi(x)+\phi(y)$ is an example of a quasiconvex set. Such graphs are known to possess many interesting measure and topological properties. These known properties and other new properties as well follow from more general results on quasiconvex sets. Let Q be a quasiconvex set; and let P be the set of those points not in Q which are interior to the convex hull of Q. Results: Q is dense in its convex hull. If Q has positive outer measure, P has zero inner measure. If Q has positive inner measure, P is null. If Q contains a nonplanar continuum, P is null. If P contains a set which together with a plane bounds a bounded open set, then Q has topological dimension zero. (Received November 11, 1948.)

147. R. G. Helsel: *Convergence in area and convergence in volume.*

Using an enclosed volume based on topological considerations, it is shown that if a sequence of closed surfaces converges in the Fréchet sense to a closed surface which occupies a point set of three-dimensional measure zero, then convergence of the Lebesgue areas implies convergence of the enclosed volumes. This result is the extension to surfaces of a theorem which Radó (Fund. Math. vol. 27 (1936) pp. 212–225) established for plane curves. The methods used are analogous to those utilized by Radó. (Received November 10, 1948.)

148*t*. M. A. Hyman: *On partial differential equations of mixed type.* Preliminary report.

The equation (I) $y^\lambda\phi_{xx}+\phi_{yy}=0$ $(\lambda>0)$ may be transformed for $y>0$ into the Beltrami-Weinstein equation (II) $\phi_{\xi\xi}+\phi_{\eta\eta}+\rho\eta^{-1}\phi_\eta=0$ $(\rho>0)$ which has been investigated recently (A. Weinstein, Trans. Amer. Math. Soc. vol. 63 (1948) pp. 342–354). A similar transformation into (II) is possible for the equation (III) $y\phi_{xx}+\phi_{yy}+\alpha y^{-1}\phi_y=0$ $(\alpha>-1/2)$. F. Tricomi (R. Academia Nazionale dei Lincei, Memorie (5) vol. 14 (1923)) has discussed equation (I) in detail when $\lambda=1$ (or $\rho=1/3$, $\alpha=0$). The present author considers the more general equations (I) and (III). Among the results obtained is a one-to-one correspondence between the family of solutions of (II) analytic across $\eta=0$ and a certain family of solutions of (I) analytic across $y=0$ (here λ is an integer); corresponding members of the two families transform into each other. A similar correspondence has been found between solutions of (II) and of (III) analytic across the respective singular lines of these equations. (Received November 19, 1948.)

149*t*. J. A. Jenkins: *Some problems in conformal mapping.*

As an extension of the concept of extremal length, introduced by Ahlfors and Beurling, modules are defined for the pentagon, hexagon and triply-connected domain. These modules are functions of several real variables and represent conformal invariants of the configurations. Various applications are made, in particular necessary and sufficient conditions are given that one triply-connected domain may be mapped conformally into another so as to have the same topological situation. These conditions relate only to the metric properties of the domains. (Received October 5, 1948.)

150*t*. R. E. Lane and H. S. Wall: *Continued fractions whose even and odd parts converge absolutely.*

A continued fraction with sequence of approximants $f_0, f_1, f_2, \cdots$ is said to converge absolutely if the series $f_0+\sum(f_p-f_{p-1})$ converges absolutely. It is shown that if the even and odd parts of $f=K_{p=0}^{\infty}(c_p/1)$ $(c_0=1)$ converge absolutely, then f converges if, and only if, the series $\sum|b_p|$ diverges, where $b_1=1$, $c_p=1/b_pb_{p+1}$, $p=1, 2, 3, \cdots$. The hypothesis holds, in particular, if there exist positive numbers r_p such that $r_1|1+c_1|>|c_1|$, $r_2|1+c_1+c_2|>|c_2|$, $r_p|1+c_{p-1}+c_p|\geqq r_pr_{p-2}|c_{p-1}|+|c_p|$, $p=3, 4, 5, \cdots$. Hence if these "fundamental inequalities" (Scott and Wall, Trans. Amer. Math. Soc. vol. 47 (1940) pp. 155–172) are satisfied, f converges if, and only if, $\sum|b_p|$ diverges. (Received November 1, 1948.)

151. R. E. Langer: *The asymptotic solutions of ordinary linear differential equations of the second order, with special reference to a turning point.*

If in an equation (1) $d^2u/dx^2+[\lambda^2q_0(x)+\lambda q_1(x)+\sum_{\nu=0}^{\infty} q_{\nu+2}(x)\lambda^{-\nu}]u=0$, with $a\leqq x\leqq b$, and λ a large parameter, the coefficient $q_0(x)$ is bounded from zero, familiar procedures are available for the derivation of asymptotic solutions. If $q_0(x)$ vanishes at a point x_0, this is called a turning-point. For an interval (a, b) which contains a turning-point the asymptotic representation of solutions of (1) in terms of elementary functions involves the Stokes' phenomenon. This latter is avoidable if the restriction to elementary functions is dropped. The forms which have heretofore been given in such terms are, however, complicated to the extent that only their leading terms are directly available. This paper develops a procedure by which the asymptotic solutions are deducible by an easy formalism, and become available to the terms in any degree of $1/\lambda$. The method is applicable whenever the turning-point is a simple zero of $q_0(x)$, and requires no conditions upon $q_1(x)$. (Received November 13, 1948.)

152*t*. Joseph Lehner: *Generalized partial fraction decompositions of functions analytic in the unit circle.*

Let $G(x)=\sum_1^\infty a_n x^n$ be regular for $|x|<1$. The author proves that a necessary and sufficient condition that $G(x)$ have a generalized partial fraction decomposition, (1) $G(x)=\sum_{k=1}^\infty b_k \sum_{l=1}^\infty \alpha_l (1-x\exp(-i\lambda_k))^{-l}$, is that (2) $a_n=g(n)f(n)$, where $g(w)$ is a polynomial in the complex variable w, or an entire function of order at most one and minimum type; $f(n)$ be a uniformly almost periodic sequence, (3) $f(n)\sim\sum_{k=1}^\infty b_k \exp(-i\lambda_k n)$, $n=1, 2, 3, \cdots$; (4) the series in (3) converge absolutely. If these conditions are fulfilled, the series in (1) converges absolutely and uniformly in every closed region of the plane which does not intersect M, the closure of the set $\{\exp i\lambda_k\}$, $k=1, 2, \cdots$; it defines an analytic function singular on M and regular on each connected piece of the complement of M. If M is the entire circle $|x|=1$, $G(x)$ has a natural boundary, whose decomposition is given by the series (1). Explicit formulae are given for the coefficients α_l. The author treats also the case in which the series in (3) is not absolutely convergent. The function $G(x)$ is still regular in the same region as before; on M it is again singular, the type of singularity at $\exp(i\lambda_k)$ being described by the asymptotic value of G under radial approach. This paper extends results of Rademacher, *A convergent series for the partition function* $p(n)$, Proc. Nat. Acad. Sci. U. S. A. (1937), and of Bochner and Bohnenblust, *Analytic functions with almost periodic coefficients*, Ann. of Math. (1934). (Received November 15, 1948.)

153*t*. G. W. Mackey: *On a theorem of Stone and von Neumann.*

Let G be a separable locally compact group. Let R be the $*$ algebra of all Borel functions on G. Let $s\to U_s$ be a representation of G by unitary operators in a Hilbert space H. Let $g\to V_g$ be a representation of R by bounded linear operators in the same Hilbert space H. The principal theorem of this paper asserts that if these representations are suitably continuous and are so related that for all $s\in G$ and all $g\in R$, $U_sV_g=V_hU_s$ where $h(x)=g(xs)$, then they are simultaneously unitary equivalent to a direct sum of replicas of the "regular representations" obtained from the Hilbert space of functions on G, square summable with respect to right invariant Haar measure, by mapping $s\in G$ into the operator $f(x)\to f(xs)$ and $g\in R$ into the operator $f(x)\to g(x)f(x)$. This theorem combined with a known result on the unitary representations of locally compact Abelian groups implies a corresponding theorem in which G is Abelian and R is replaced by the character group of G. If G is taken to be the n-dimensional vector group this last theorem reduces to a theorem of Stone and von Neumann asserting the essential uniqueness of operators satisfying the Heisenberg commutation relations. (Received November 16, 1948.)

154*t*. G. W. Mackey: *On semi-direct products of locally compact Abelian groups*. I. Preliminary report.

Let G be a separable locally compact group. If G admits closed Abelian subgroups G_1 and G_2 such that G_1 is normal, $G_1\cap G_2=e$ (the identity) and $G_1\cdot G_2=G$ then G is said to be a semi-direct product of the locally compact Abelian groups G_1 and G_2. This paper is the first of a projected series in which it is proposed to study the properties of such semi-direct products in detail. In it the unitary representations of these groups are studied. All finite-dimensional irreducible unitary representations are explicitly determined. For the members of a certain class of semi-direct products (including the "$ax+b$ group" recently studied by Gelfand and Neumark) the infinite irreducible unitary representations are also determined. It is shown by several types of examples that this last determination does not apply to all semi-direct products. (Received November 16, 1948.)

155. G. R. MacLane: *Rational functions with all zeros and poles on a rectifiable Jordan curve.*

The following theorem is proved: let $f(z)$ be holomorphic and never zero in a simply-connected domain D of the z-plane bounded by a rectifiable Jordan curve C. There exists a sequence of rational functions $R_n(z)$, each of which has all of its zeros and all of its poles on C, and such that $\lim_{n\to\infty} R_n(z)=f(z)$ for z in D, uniformly in any closed subset of D. The condition on $R_n(z)$ is for the complete plane; that is, $R_n(z)$ is regular and doesn't vanish at $z=\infty$. (Received November 5, 1948.)

156. Szolem Mandelbrojt: *A converse theorem on approximation on the real axis.*

The author has proved a theorem giving conditions in order that, $F(x)>0$ and $\{K_n\}$ (positive integers) given, to each $\epsilon>0$ there correspond a polynomial $P(x)$ of the type $P(x)=a_0+a_1x^{K_1}+\cdots+a_nx^{K_n}$ such that $|f(x)-P(x)|<\epsilon F(x)$ $(-\infty<x<\infty)$. This is a generalization of a theorem of S. Bernstein. The author proves now that the above-mentioned theorem cannot be, in a certain sense, much improved. (Received November 15, 1948.)

157t. F. I. Mautner: *On maximal Abelian subalgebras of operator algebras.* I.

The maximal comutative self-adjoint subalgebras of weakly closed self-adjoint operator algebras W in a Hilbert space H are of decisive importance for the decomposition and structure of W. In the present paper the following theorem is proved for separable Hilbert spaces H: Let C be any maximal self-adjoint commutative subalgebra of W and P any maximal self-adjoint commutative subalgebra of W', where W' consists of all those bounded operators in H which commute with every element of W. Assertion: The weakly closed self-adjoint operator algebra generated by C and P is a maximal self-adjoint commutative subalgebra of the algebra of all bounded operators in H. A similar property has been formulated by W. Ambrose in a Princeton seminar for slightly different algebraic systems, namely those arising from the L_2-systems of groups. This proof uses generalized decomposability of H into irreducible spaces and also von Neumann's generalized decomposability of W into factors. (Received November 12, 1948.)

158. C. N. Moore: *On the functional behavior of a function defined by a certain Dirichlet's series.* II.

The study initiated in a paper presented to the Society at the meeting in New York in October, 1948, is here continued. It is proved that the function $L^{(c)}(s)$, defined in the previous paper, has the asymptotic properties needed for its application to the problem of prime pairs. (Received November 16, 1948.)

159. O. M. Nikodým: *On the spectrum of measurable functions and on equimeasurable transformations.*

By a "soma" (Carathéodory's term) belonging to a measurable set a is understood the class of all measurable sets, b, which are almost equal to a, that is, meas. $(b-a)$ plus meas. $(a-b)=0$. We shall consider Boolean fields (measure algebras) whose elements are somata. To a given finite measurable function $f(x)$ defined on a set I there corresponds a Boolean field (T), the smallest one containing all the somata belonging to $[E(1)]$ where $[E(1)]$ is the set of all the x for which $f(x)$ does not exceed 1. (T) will be termed "spectral field of $f(x)$." Theorems on subfields of (T) in connection with functions $G(f(x))$ where $G(1)$ is a borelian function will be presented. The main question to be discussed is: whether $f(x)$ can be carried by an almost one-to-one and equimeasurable transformation into a monotonous function. Conclusive answer is given in the form of a general theorem. The proof is based on a deep lemma on Boolean fields connected with the known Maharam's Theorem, and on J. von Neumann's theorem on equimeasurable transformations. There are some relations to the multiplicity of the spectrum of Hermitian transformations in Hilbert space. (Received December 13, 1948.)

160. H. C. Parrish: *Note on certain decompositions of point sets.*

This paper concerns an elementary instance of "*methodical generalization*," that is, generalization by means of generally known cues. An immediate generalization of a well known theorem, namely, that a linear set is the disjoint sum of a dense-in-itself and a scattered set, leads to the problem of the genesis of an unconditionally additive set property α, that is, one such that the sum of any number of sets of property α has property α. Every such α yields an analogue of the particular case. A complete genesis of unconditionally additive set properties is obtained in terms of ascending set-point properties. (A set-point property $\beta \equiv \beta(E, x)$, that is, a property of a set E in relation to a point x, is ascending if its validity for (E, x) implies its validity for (E_1, x) when $E_1 \supset E$.) It is then shown that ascending set-point properties may be generated from "ascending set-interval properties," and the latter, in turn, from "ascending set properties"—the meaning of the latter terms being manifest. The results unify the literature analogues of the particular theorem, and add new cases. Each particular case gives a geometric property of unconditioned sets and of unconditioned functions. (Received December 9, 1948.)

161. M. O. Reade: *On p-analytic functions.*

Let $f(z)$ be defined in a bounded, simply-connected domain $\mathcal{D}$. Kriszten has defined $f(z)$ to be p-analytic in $\mathcal{D}$ if and only if $A^p f(z)=0$ holds in $\mathcal{D}$; here p is a fixed positive integer and A is the complex differential operator $\partial/\partial x + i\partial/\partial y \equiv A$ (Comment. Math. Helv. vol. 21 (1948) p. 73). In this paper, integral characterizations for p-analytic functions in general, and for certain p-analytic polynomials in particular, are derived; these results extend earlier results due to Féderoff (Rec. Math. (Mat. Sbornik) vol. 41 (1934) p. 92), Haskell (Bull. Amer. Math. Soc. vol. 52 (1946) p. 332), and Reade (Bull. Amer. Math. Soc. vol. 53 (1947) p. 98). The following results are typical ones. (I) Let $f(z)$ be continuous in $\mathcal{D}$, let $C(z_0, r)$, for sufficiently small r, denote the circle in $\mathcal{D}$ center at z_0 and radius r, and let $p \geqq 0$. A necessary and sufficient condition that $f(z)$ be $(p+2)$-analytic in $\mathcal{D}$ is that $F_r(z) \equiv \int_{C(z,r)} \zeta^p f(z+\zeta)d\zeta$ be 1-analytic for all sufficiently small r, in the subdomain $\mathcal{D}_r$ where $F_r(z)$ is defined. (II) Let

Π_n denote a regular n-gon, and let n be a fixed positive integer, $n \geqq 2p+2$. A necessary and sufficient condition that (i) $f(z)$ be $(p+1)$-analytic in $\mathcal{D}$ and (ii) $\bar{A}^{(n-p-1)}f(z) \equiv 0$, where $\bar{A} \equiv \partial/\partial x - i\partial/\partial y$, is that $\int_{\Pi_n} \zeta^p f(\zeta) d\zeta = 0$ for all Π_n in $\mathcal{D}$. (Received November 16, 1948.)

162*t*. M. O. Reade: *Sufficient conditions that $f(z)$ be analytic.*

Let $D_1, D_2, \cdots, D_n, \cdots$ be a sequence of simply-connected domains, each bounded by a simple, closed, rectifiable curve, such that (i) $\int\int_{D_n} z dx dy = \int\int_{D_n} z^2 dx dy = 0$, $n = 1, 2, 3, \cdots$, (ii) for each $\epsilon > 0$ there exists $m = m(\epsilon)$ such that D_n is contained in $|z| \leqq \epsilon$, for $n > m$. Now if $f(z)$ is continuous in a bounded, simply-connected domain $\mathcal{D}$, then a necessary and sufficient condition that $f(z)$ be analytic in $\mathcal{D}$ is that $\lim_{n\to\infty} [1/|D_n|^2 \int\int_{D_n} z f(z_0+z) dx dy] = 0$ for each z_0 in $\mathcal{D}$; here $|D_n|$ denotes the area of D_n. This generalizes a theorem due to Fédoroff (Rec. Math. (Mat. Sbornik) vol. 41 (1934) p. 92). (Received November 16, 1948.)

163. L. V. Robinson: *A new operational method for solving partial differential equations.* Preliminary report.

This method makes use of an operator, $\partial/\partial x + P(x, y)\partial/\partial y$, and its inverse, to solve linear partial differential equations, of the first and second orders, and some nonlinear equations. (Received November 16, 1948.)

164. P. C. Rosenbloom: *On the iteration of entire functions.*

By a fixed point of a function f we mean a root of the equation $f(z) = z$. If f is an entire function and if $f(f(z))$ has only a finite number of fixed points, then f is a polynomial. More generally, if f and $f(g(z))$ have only a finite number of fixed points and if f is a transcendental entire function, then $g \equiv z$ or $g \equiv$ constant. If f_n, the nth iterate of f, has only a finite number of fixed points, and $n > 1$, then f is a polynomial. The proofs are based on Nevanlinna's second fundamental inequality, on an inequality of Bohr in a precise form, and on Robinson's numerical estimate in Schottky's theorem. (Received December 2, 1948.)

165. Robert Schatten: *"Closing-up" of sequence spaces.*

Let $\mathfrak{L}$ denote the linear set of sequences of real numbers $(a_1, a_2, \cdots)$ having only a finite number of nonzero terms. A symmetric norm on $\mathfrak{L}$ is a function $\phi(a_1, a_2, \cdots)$ subject to conditions: (i) $\phi(a_1, a_2, \cdots) > 0$ unless $a_1 = a_2 = \cdots = 0$, (ii) $\phi(\alpha a_1, \alpha a_2, \cdots) = |\alpha| \phi(a_1, a_2, \cdots)$ for any constant α, (iii) $\phi(a_1+b_1, a_2+b_2, \cdots) \leqq \phi(a_1, a_2, \cdots) + \phi(b_1, b_2, \cdots)$, (iv) $\phi(a_1\ a_2, \cdots) = \phi(\epsilon_1 a_{1'}, \epsilon_2 a_{2'}, \cdots)$ where $\epsilon_i = \pm 1$ and $1', 2', \cdots$ denotes any permutation of $1, 2, \cdots$. It follows that $\rho(a_1, \cdots, a_n, 0, 0, \cdots)$ is a nondecreasing function of n. Consequently, $\phi(a_1, a_2 \cdots)$ determines in the usual fashion a conjugate norm $\psi(a_1, a_2, \cdots)$. Thus, we obtain two normed linear spaces $\mathfrak{L}(\phi)$ and $\mathfrak{L}(\psi)$. We define $[[\mathfrak{L}(\phi)]]$ as the Banach space of all infinite sequences $(a_1, a_2, \cdots)$ of which an infinite number of a_i may be $\neq 0$, for which $\lim_{n\to\infty} \phi(a_1, \cdots, a_n, 0, 0, \cdots) < +\infty$, while $[\mathfrak{L}(\phi)]$ is defined as the Banach space of all fundamental sequences of elements in $\mathfrak{L}(\phi)$. The space $[\mathfrak{L}(\phi)]$ is quite often a *proper subset* of $[[\mathfrak{L}(\phi)]]$, as for instance, in the case $\phi(a_1, a_2, \cdots) = \max a_i$. The conjugate space of $[\mathfrak{L}(\phi)]$ may be characterized as $[[\mathfrak{L}(\psi)]]$. (Received November 10, 1948.)

166. H. M. Schwartz: *Notes on the differential equation* $y''(x) - h(x, \lambda)y(x) = 0$. I. Preliminary report.

The present note is concerned with the following question relating to the differential equation $y''(x) - h(x, \lambda) = 0$ defined in the x-interval $(0, \infty)$: If $h(x, \lambda)$ is for $x > 0$ a regular analytic function of λ in a given domain D of the complex λ-plane, but is not finite at $x = 0$, what conditions on h will insure the regularity in the domain D of the solution of the differential equation for which $y(0, \lambda) = 0$, $y'(0, \lambda) = 1$? By the use of standard methods a number of such conditions are established of which the following is typical: If the coefficient h is such that the solution in question exists, then a sufficient condition for the required result is the existence of the integral $\int_0^a (x \cdot h)^2 dx$ and its boundedness uniformly with respect to λ in D, for every fixed $a > 0$. (Received November 17, 1948.)

167. Seymour Sherman: *On a problem of Raikov.*

Raikov (*Harmonic analysis on commutative groups with the Haar measure and the theory of characters*, Trav. Inst. Math. Stekloff vol. 14 (1945) 86 pp.), for the purpose of harmonic analysis, presents a postulational development of Haar-Raikov measure on topological groups. The domain of this measure function is the σ-ring of all strict Borel sets. It is not obvious that continuous functions are measurable with respect to this σ-ring. Raikov raises the question of extending this invariant measure to all Borel sets. This is done here and in answer to another of Raikov's questions it is shown that the relation between the extended measure on the group and the extended measure on its completion is the natural one. It is also shown that the σ-ring of σ-bounded strict Borel sets is the same as the σ-ring of Baire sets. A fundamental tool used in settling Raikov's first question is the recently announced result of Halmos: Haar measure is completion regular. (Received November 4, 1948.)

168. G. E. Springer: *Distortion theorems in pseudoconformal mapping.*

In order to obtain new domains of comparison needed for the evaluation of bounds in pseudoconformal (p.c.) mapping, the p.c. properties of several special domains in the space of two complex variables are considered; namely, $E_\lambda: |z_1|^2 < e^{-\lambda |z_2|^2}$, $\lambda > 0$, and $C_A: |z_1|^2 < 1 - |z_2|^2 - A|z_2|^4$, $A > 0$. Complete systems of orthonormal functions $\{\phi_\nu(Z)\}$, $Z = (z_1, z_2)$, are determined for E_λ and C_A and are used to define the kernel function $K(Z, \bar{Z}) = \sum_\nu \phi_\nu(Z)\overline{\phi_\nu(Z)}$ and the p.c. invariant $I(Z) = (1/K(Z, \bar{Z})) \cdot (T_{1\bar{1}}T_{2\bar{2}} - |T_{1\bar{2}}|^2)$, where $T_{m\bar{n}} = \partial^2 \log K(Z, \bar{Z})/\partial z_m \partial \bar{z}_n$ (Bergman, Mémorial des Sci. Math. vol. 106 (1947)). 1. In any p.c. transformation of E onto itself each shell $|z_1|^2 e^{\lambda |z_2|^2} = \text{constant}$ is mapped into itself. 2. If E is mapped by a schlicht p.c. transformation onto a domain D such that $E_{\lambda_1} \subset D \subset E_{\lambda_2}$, the image W^0 of any point Z^0 of E_λ lies between certain shells $|z_1|^2 e^{\mu |z_2|^2} = c$, where μ and c depend only upon λ, λ_1, λ_2 and Z^0. 3. Each value of A determines a different p.c. class of which C_A is a representative. 4. In any map of C_A onto itself, the origin is a fixed point. 5. If C_A is mapped onto a domain B such that $C_{A_1} \subset B \subset C_{A_2}$, the image of the origin lies in a hypersphere with center at the origin and radius depending only upon A, A_1 and A_2. (Received November 12, 1948.)

169. Otto Szasz: *On some trigonometric transforms.*

In previous papers [see Ann. of Math. vols. 43 and 46; Trans. Amer. Math. Soc. vol.

54; Duke Math. J. vol. 11] some trigonometric transforms and their application to Fourier series were discussed. The present paper contains further results, in particular for the series to sequence transform $A_n = \sum_{v=1}^{n} U_v \sin v\theta/v\theta$, where $\theta = \theta_n$ decreases to zero as $n \to \infty$. A detailed discussion of this transform was stimulated by a forthcoming paper of C. Lanczos who came to consider the same transform from a different point of view. Furthermore, related transforms are discussed. The main results concern the comparison of the author's transform with the Cesàro scale. (Received November 13, 1948.)

170. H. P. Thielman: *On a class of singular integral equations occurring in physics.*

The Wiener-Hopf integral equation is solved by elementary means for the case when the kernel $K(x, y)$ is a function of the absolute value of $x-y$ only, and satisfies the same linear differential equation with constant coefficients in each of the regions $0 \leqq y \leqq x$, $0 \leqq x \leqq y$. (Received November 15, 1948.)

171. J. L. Ullman: *A simplified proof of Jentzsch's theorem.* Preliminary report.

Let $g(z) = \sum_0^\infty a_n z^{-n}$, $a_0 = 1$, $\limsup |a_n|^{1/n} = 1$, $g_k(z) = \sum_0^k a_n z^{-n} = \prod_1^k (1 - z_{ki} z^{-1})$, then Jentzsch's theorem states that if $|z_0| = 1$, z_0 is a limit point of the set $\{z_{ki}\}$, $i \leqq k$, $k = 1, 2, \cdots$. The proof of the case where $g(z) \neq 0$ for $|z| > 1$ illustrates the method. The Cauchy-Hadamard formula applied to the expansion of $g(w)/(w-z)$ in powers of $1/w$ yields $\limsup |z^n g_n(z)|^{1/n} = 1$ if $|z| \leqq 1$. A contradiction arises by assuming that a point of unit modulus, say 1 without loss of generality, is not a limit point of $\{z_{ki}\}$. It follows by Hurwitz' theorem that Re $(z_{ni}) < \rho < 1$ for some $\rho < 1$ and $n > N_\rho$. But then it follows for these values of n that $|b - z_{ni}| > \eta |a - z_{ni}|$ for some $\eta > 1$ and $\rho < a < b < 1$. Hence the contradiction $\limsup |b^n g_n(b)|^{1/n} > \limsup |a^n g_n(a)|^{1/n}$. The same method applied to the generating function of Faber polynomials leads to new results concerning the asymptotic distribution of the zeros of these polynomials. (Received November 12, 1948.)

Applied Mathematics

172*t*. Milton Abramowitz: *Asymptotic solutions of the spheroidal wave equation.*

Solutions of the spheroidal wave equation $(1-z^2)w'' - 2(m+1)zw' + (b - c^2z^2)w = 0$ are obtained for the cases (1) m large and c small (2) m large and c large. In the first instance, the solutions are expressed in terms of Hermite polynomials while in the second they are expressed in terms of parabolic cylinder functions. In both cases approximations are obtained for the separation constants b and some numerical results are cited to illustrate the formulas. (Received October 13, 1948.)

173. R. Bott and R. J. Duffin: *On the algebra of networks.* Preliminary report.

An electrical or mechanical system is considered having a quadratic energy law $E = \sum g_i v_i^2 = \sum \sum A_{rs} q_r q_s$. The v_i are constrained coordinates, linear functions of unconstrained coordinates q_r. It is shown that $D = \det A_{rs}$ is a homogeneous multilinear form with positive coefficients as a function of the parameters g_i. For example, if the parameters are the conductivities of the wires in a direct current network, the terms of D are the Cayley tree products. If $\chi = \log D$, the impedance conjugate to the parameter g_i is $\chi_i = \partial\chi/\partial g_i$. This suggests that χ be termed the *impedance potential*. If all g_i have positive real part, so also do all χ_i. The second derivatives χ_{ij} are negative squares of rational functions R_{ij}. The transfer impedance matrix R_{ij} has many interesting properties. The quantities derived from the impedance potential are invariants not depending on the choice of the coordinate system q_r. Conversely, it is shown that the logarithm of any homogeneous multilinear function is the impedance potential of some system if its second derivatives are negative perfect squares. This partially confirms a conjecture by Richard Cohn. (Received November 16, 1948.)

174. Y. W. Chen: *Initial value problems of a hyperbolic system of quasi-linear differential equations with singularity.*

When initial values on the line $x+y=0$ or on the two half-axes $x \geqq 0$, $y=0$; $y \geqq 0$, $x=0$ are properly given, solution of the equation $L(u;k,l) \equiv u_{xy} + k(x+y)^{-1}(u_x+u_y) + l(x+y)^{-2}u = 0$ with constants, k, l, can be obtained by Riemann's method. With the help of Riemann function an iteration scheme can be set up to obtain solution of initial or characteristic initial value problem of equation $L(u;k,l) = f(x,y;u,u_x,u_y)$, assuming proper behavior of the function f on the right-hand side. Suppose we have a hyperbolic system of quasi-linear differential equations $\sum_{j=1}^{n} a_{i,j} u_x^j = 0$, $\sum_{j=1}^{n} a_{h,j} u_y^j = 0$, $i = 1, 2, \cdots, n'$ $(<n)$, $h = n'+1, \cdots, n$. The values of u^j assigned on $x+y=0$ or on the two half-axes $x \geqq 0$, $y=0$; $y \geqq 0$, $x=0$ have the property that after substituting them into the coefficients $a_{s,j}$, $s=1, 2, \cdots, n$; $j=1, 2, \cdots, n$, the determinant $\|a_{s,j}\|$ becomes zero on the initial line or at the vertex $(x, y) = (0, 0)$. Under certain conditions such a system of equations can be reduced to another system of n equations of the following form: $j=1, 2, \cdots, r$, $L(u^j; k_j, l_j) = f_j(x, y; u^1 \cdots u^n, u_x^1 \cdots u_x^n, u_y^1 \cdots u_y^n)$, where k_j and l_j are constants. The initial value problems can then be treated by iteration with the help of Riemann functions. In particular this method can be applied to certain problems of supersonic axial symmetric flows, which were mathematically formulated and investigated by K. O. Friedrichs. (Received November 16, 1948.)

175*t*. R. J. Duffin: *Nonlinear networks*. IV.

The networks considered in this paper consist of transformers with ferromagnetic cores (assumed to have negligible hysteresis) and Ohmic resistors. It is found that as in the case of linear networks a given electromotive force gives rise to a unique state of currents after sufficient time has elapsed. Moreover, if the electromotive force is periodic, a unique periodic current flow can exist. The proof depends upon transforming the network equations into equations for a network containing nonlinear resistors. The differential permeability of a ferromagnetic lies between positive limits. In the transformed equations this is imaged by the condition that the differential resistance lies between positive limits. The results stated then follow from theorems previously developed for networks containing such quasi-linear resistors. (Received November 16, 1948.)

176*t*. W. F. Eberlein: *On the convergence of certain perturbation series*. Preliminary report.

Despite decades of numerical evidence suggesting that the radii of convergence ρ_n of the perturbation series $\lambda_n(\epsilon) = \lambda_{0n} + a_{1n}\epsilon + a_{2n}\epsilon^2 + \cdots$ for the characteristic values λ_n

of the Mathieu equation $u''(x)+(\lambda+\epsilon\cos 2x)u(x)=0$, $u(0)=u(2\pi)$, and related wave equations are nonzero and increase with the natural index n, the only published result appears to be that of G. N. Watson (1915). The latter obtained a positive lower bound for ρ_0 of the Mathiew equation by a classical method which did not extend to higher characteristic values. It is shown that the modern perturbation theory of operators in Hilbert space developed by Rellich and de Sz. Nagy (Comment. Math. Helv. vol. 19 (1947)) yieldsexplicit lower bounds of the form $\rho_n \geqq Kn$ $(K>0)$ in problemsof this type. (Received November 18, 1948.)

177. G. E. Forsythe: *A solution of the telegrapher's equation with initial conditions on only one characteristic.* Preliminary report.

When conditions (c) and (ii) below are satisfied, the future motion in a certain linearized model (due to Rossby) of non-divergent, plane atmospheric flow is uniquely determined by the present motion. We have the following unusual boundary-value problem with initial conditions on one characteristic: Let $f(x)$ be a real-valued function of bounded variation for $-\pi \leqq x \leqq \pi$, such that: (a) $f(-\pi)=f(\pi)$, (b) $f(x)$, $=[f(x+0)+f(x-0)]/2$, (c) $\int_{-\pi}^{\pi} f(x)dx=0$. The problem is to define $v(x, t)$ for $-\pi \leqq x \leqq \pi$ and $0 \leqq t < \infty$ such that: (i) the telegrapher's equation $\partial^2 v/\partial x \partial t + v/4 = 0$ is satisfied for all t and almost all x; (ii) $v(-\pi, t) = v(\pi, t)$; (iii) $v(x, 0) = f(x)$. It is shown that the unique solution of the problem is given by $v(x, t) = \sum_{n=1}^{\infty} (a_n \cos nz_n + b_n \sin nz_n)$, where $z_n = x + (t/4n^2)$, and where a_n, b_n are the Fourier's coefficients of $f(x)$. A Green's function is defined, $G(x, t) = \sum_{n=1}^{\infty} (1/n)\sin nz_n$, and it is shown that v can always be represented by a Lebesgue-Stieltjes integral of G: $v(x, t) = (1/\pi)\int_{-\pi}^{\pi} G(x-u, t)df(u)$. A program is outlined to compute $G(x, t)$ to three decimal places for $x = -\pi(\pi/36)\pi$ and $t=0(36)144$, using 10-decimal-place calculating equipment. Computing G from the defining series would use about 92,000 terms and require about 26,500,000 multiplications. Improvement of the convergence along lines suggested by Tamarkin reduces the number of multiplications to about 12,000. The hardest feature of the program is to avoid exceeding 10-place machine capacity. (Received November 15, 1948.)

178. Herbert Jehle: *Nonlinear resonance and self-duplication of genes.*

The current view about self-duplication is that the macromolecules which a gene is composed of attract from the surrounding medium specifically those macromolecules which happen to be identical with the former ones. This selection is extremely accurate, and this phenomenon's versatility which allows all kinds of genes and their mutants to self-duplicate is equally remarkable. The macromolecules are known to be rigid. Most of their thermal vibrations have so densely spaced quantum levels that they can be treated classically. Let "modes" be vibrations which are normal for very small amplitudes. Nonlinearity, that is, cubic and quartic coupling terms in the potential energy, will make synchronization possible for sets of almost commensurable modes of a molecule. In other words, librational motion about definite phaserelations of a set of commensurable frequencies can occur, and therefore there will be a statistical preference for certain phaserelations. A pair of macromolecules interacts through electric dipole vibrations accompanying their mechanical ones. If the pair of fundamental modes of a pair of identical macromolecules is inphase, then also the pairs of other corresponding modes are mostly inphase. This means a total interaction energy which can become larger than kT, and therefore almost exclusively inphase vibration, and sufficient attraction even at long range. (Received November 16, 1948.)

179t. R. E. Langer: *A note on the theory of vibration-rotation interaction.*

The equation for the radial component F of the *eigenfunction* of a rotating harmonic oscillator is of the form $d^2F/dr^2-[(r-1)^2/4\alpha^2+E/\alpha\omega_e+l(l+1)/r^2]F=0$. The constant α is small, so that $1/\alpha$ appears as a large parameter. l is an integer. The problem is to determine the characteristic values of E for which the equation admits of a solution that vanishes both at $r=0$ and at $r=\infty$. This requires special considerations of the solutions of the equation at the singular point $r=0$ and around the turning-point $r=1$. The characteristic E-values are determined to within terms of the order of $\alpha \log \alpha$, and the characteristic solutions are described in the various intervals of the range $(0, \infty)$. (Received November 13, 1948.)

180. C. G. Maple: *The Dirichlet problem: bounds at a point for the solution and its derivatives.*

Synge has shown that the vector representation in function space of the solution of the Dirichlet problem must lie on a hypercircle in the function space. For a suitably defined free Green's vector, bounds for the scalar product of the solution vector and the Green's vector are established by making use of this result and the fact that the metric of the function space is positive definite. This same scalar product is then expressed in terms of certain calculable integrals and the value of the solution at a point by use of a well known harmonic mean value theorem. When these results are combined, bounds at a point interior to the region of definition of the problem for the solution are obtained. By a change in the definition of the Green's vector, bounds at an interior point are established for the derivatives of the solution. When the region of definition of the problem is such that the bounding surface contains a plane portion, bounds at a point on this plane portion for the normal derivative are established. (Received November 15, 1948.)

181t. George Pólya and Gabor Szegö: *Approximations and bounds for the electrostatic capacity and for similar physical quantities.*

This paper consists of eight chapters: 1. Definitions, methods and results. 2. The principles of Dirichlet and Thomson. 3. Applications of the principles of Dirichlet and Thomson to estimation of the capacity. 4. Circular plate condenser. 5. Torsional rigidity and principal frequency. 6. Nearly circular and nearly spherical domains. 7. On symmetrization. 8. On ellipsoid and lens. Besides bringing proofs for, and complements to, some results announced elsewhere (especially Amer. Math. Monthly vol. 54 (1947) pp. 201–206) the paper presents in a new form the second variation of certain important functionals when the domain varied is a circle or a sphere (chap. 6), gives new inequalities between torsional rigidity, principal frequency and the maximum inner radius of simply connected plane domains (chap. 5), brings extensive surveys of solved and open problems (chap. 1), and so on. (Received November 1, 1948.)

182t. George Pólya and Alexander Weinstein: *On the torsional rigidity of multiply connected cross sections.*

Of all multiply connected cross sections with given area and with given joint area of the holes, the ring bounded by two concentric circles has the maximum torsional rigidity. This theorem generalizes a statement due to de Saint-Venant, which has been proved recently by one of the authors jointly with G. Szegö. The same method is used in proving the present theorem in conjunction with a reshaping of the theory of torsion for multiply connected cross sections. (Received November 1, 1948.)

183*t*. Eric Reissner: *Note on compressibility corrections for subsonic flow past slender bodies of revolution.*

The relation between two-dimensional and axi-symmetrical compressibility corrections for flow past slender bodies, according to the linear perturbation theory, is examined for the problem of flow past a corrugated cylinder. A solution in terms of modified Bessel functions is obtained which contains as limiting cases both the Prandtl-Glauert correction and the Göthert correction and which moreover shows the nature of the transition from one to the other of the two limiting cases. In the present problem the form of the Göthert correction which previously has caused some discussion (see for instance W. R. Sears, Quarterly of Applied Mathematics vol. 5 (1947) pp. 89–91) is obtained in particularly simple fashion. (Received November 5, 1948.)

184*t*. Eric Reissner: *On the theory of thin elastic shells.*

The present paper gives a self-contained formulation of the problem of *finite* symmetrical deflections of shells of revolution. From this there is obtained by specialization the small-deflection (linearized) theory. In the reduction of this latter theory to two simultaneous second order differential equations slight modifications of the known results are introduced, the advantages of which are indicated. From the equations of the small-deflection theory there is derived a simplified system of equations which applies for *shallow* shells. It is shown that the solutions of these simplified equations can be expressed in terms of Bessel functions for the class of paraboloidal shells of constant thickness and that solutions in terms of elementary functions are obtained for a practically interesting class of shallow shells of varying thickness. (Received November 15, 1948.)

185*t*. H. E. Salzer: *Formulas for numerical differentiation in the complex plane.*

Formulas are given for numerical differentiation of a complex analytic function $f(z)$, where $z=x+iy$. If h is the length of the square in the Cartesian grid, $z_j=z_0+jh$, differentiation of Lagrange's n-point formula yields $h^\nu f^{(\nu)}(z_j)\sim\sum_k L_k^{(\nu)}(j)f(z_k)$. (All expressions, such as $L_k^{(\nu)}(j)$, are understood to depend upon n.) The summation is over the n fixed points z_k which are chosen to lie, for each n, in a close configuration in the z-plane, resulting in better approximation to $f^{(\nu)}(z)$ than the simpler method of choosing z_j to lie equidistant on a straight line parallel to either the x- or y-axis. Exact values of $L_k^{(\nu)}(j)$ are given for the three- to nine-point cases, for obtaining all the derivatives $f^{(\nu)}(z_j)$, $\nu=1, 2, \cdots, n-1$, at each of the n points z_j. The computation was expedited by two new formulas for $L_k^{(\nu)}(j)$, namely the recursion formula $L_k^{(\nu)}(j)=\nu[A_kL^{(\nu)}(j)/\nu-L_k^{(\nu-1)}(j)]/(j-k)$, for $k\neq j$, and $L_k^{(\nu)}(j)=A_jL^{(\nu+1)}(j)/(\nu+1)$, for $k=j$. Here $L^{(\nu)}(j)=d^\nu L(p)/dP^\nu|_{p=j}$ where $L(p)\equiv\prod_j (P-j)$ and $A_k\equiv 1/\prod_j' (k-j)$ ($\prod_j$ for all points j; $\prod_j'$ for all points except $j=k$.) For configurations that are symmetric about the 45° ray, $L_{b+ia}^{(\nu)}(d+ic)\overline{L_{a+ib}^{(\nu)}(c+id)}/i^\nu$. (Received November 10, 1948.)

186*t*. B. R. Seth: *Bending of an elliptic plate with a confocal hole.*

The bending of an elliptic plate with a confocal hole subjected to uniform pressure and clamped at the edges is discussed. The numerical results obtained are compared with those for a complete plate. It is found that the maximum deflection, W_m, occurs near the inner boundary. For the plate bounded by ellipses whose semi-axes are (1.6c, 1.249c) and (1.14c, 0.548c), it is found that W_m is almost one-fifth the value for the plate complete up to the outer boundary. By making the minor axis of the hole vanishingly small, the interesting case of an elliptic plate with a crack can be treated. (Received November 16, 1948.)

187. A. C. Sugar: *An axiomatic development of classical mechanics.* Preliminary report.

In an earlier paper the writer gave a rigorous definition of dimensional number by set theoretic methods and, in terms of this concept, defined the notion of physical function. Employing this theory of physical number and function an axiomatic reconstruction of classical mechanics is made without a direct or indirect use of the notion of an actual infinitesimal. The laws of motion are formulated for bodies rather than particles thereby avoiding the usual pseudo-derivation of body dynamics from particle dynamics. (Received December 16, 1948.)

188. C. A. Truesdell: *The kinematics of vorticity.* I.

This paper contains a general treatment of those aspects of the kinematics of continuous media which bear on the vorticity of fluid motions. The presentation is in terms of vectorial and dyadic formulae, and full historical references are given. Section I contains necessary preliminaries regarding Lagrangian and Eulerian descriptions of the motion; rate of change formulae; line, surface, and volume integrals; trajectories and tubes. Section II contains an analysis of a general motion of a continuous medium, in which the vorticity represents one part; vortex-lines, vortex-tubes, and various vorticity integrals are introduced. Section III is devoted to convection and diffusion of vorticity. These concepts are first introduced from a Eulerian viewpoint; then it is shown that their meaning can be fully clarified by a Lagrangian analysis, generalizing a result of Cauchy. The effect of the mechanism of diffusion upon the circulation, the vortex-lines, and the strength of the vortex-tubes is analyzed in detail, both from a Eulerian and from a Lagrangian point of view. (Received November 15, 1948.)

189*t*. Arthur Zeichner: *Certain singularities of the compressibility equation.*

The author considers solutions of the compressibility equation, $\psi_{\eta\eta}+l(\eta)\psi_{\theta\theta}=0$, $l(\eta)=-\eta+\sum_{n=2}^{\infty} c_n\eta^n$, $\eta(M)$ being a regular function of the Mach number M which is regular at $M=1$; $\eta(\lambda)=0$. With the use of the integral operator method (see Bergman, Amer. J. Math. vol. 70 (1948)), the behavior of multivalued solutions which possess a singularity on the line $M=1$ is investigated. Singularities at the point $M=1$, $\theta=0$ for the simplified equation, that is, where $l(\eta)=l^*(\eta)=-\eta$ (Tricomi equation), are also considered. By setting $f=\zeta^{1/2}$, in the operator $\mathrm{II}(f)$ introduced by Bergman ((4.11), loc. cit.) and applying the Kummer transformation for hypergeometric functions, the solution $\psi=A\eta(\theta^2-4\eta^3/9)^{-1/6}F(1/6, 2/3, 4/3; (-4\eta^3/9)(\theta^2-4\eta^3/9)^{-1})$ $+B(\theta^2-4\eta^3/9)^{1/6}F(-1/6, 1/3, 2/3; (-4\eta^3/9)(\theta^2-4\eta^3/9)^{-1})$ is obtained. Here A and B are constants, and $F(\cdots)$ denotes the hypergometric function. Analogous expressions are obtained for $f=\zeta^{\nu/2}$, ν positive integers. The fundamental solution, with a logarithmic singularity at M_0, θ_0, is derived for $M_0\leqq 1$. (Received November 18, 1948.)

Geometry

190. N. A. Court: *Semi-inverse tetrahedrons.*

Two tetrahedrons $(T)=ABCD$, $(T')=A'B'C'D'$ whose corresponding vertices A, A'; $\cdots$ are pairs of inverse points with respect to a sphere (M) are said to be "semi-inverse" for (M). Among the properties of two such tetrahedrons the following may be mentioned. The two tetrahedrons correspond to each other in the homology $(M, \mu, -1/c)$ having for center the center M of (M), for plane of homology the common radical plane μ of the sphere (M) and the circumspheres (O), (O') of (T), (T'), and for constant of homology the negative reciprocal of the ratio c of the power of M for (O) to the square of the radius of (M). The circumcircles of two corresponding faces of (T), (T') lie on a sphere whose center is the mid-point of the segment determined by the poles, for (M), of the two faces considered. The tetrahedral polar planes of M for (T), (T') are coaxal with the plane μ. The pedal tetrahedron of M for one of the two tetrahedrons (T), (T') and the antipedal tetrahedron of M for the other are two semi-inverse tetrahedrons with respect to (M). (Received November 15, 1948.)

191. Michael Goldberg: *Rotors in a regular tetrahedron.*

A rotor of a polyhedron is defined here as a convex surface (not necessarily spherical) which may be rotated through every possible orientation while keeping in continual contact with all the faces of the fixed polyhedron. The well known surfaces of constant width are rotors for the parallelepipeds which include the cube as a special case. This paper exhibits a class of rotors for the regular tetrahedron. These rotors are described by the polar tangential equation $p=a+bl^2+cm^2$ where p is the distance from the origin to the tangent plane and l, m are the direction cosines of the normal to the tangent plane. Other geometric properties of these rotors are derived. (Received October 25, 1948.)

192. A. J. Hoffman: *On the foundations of inversion geometry.*

The author gives an autonomous development of inversion geometry over certain kinds of fields F, including the case in which F is the real numbers. The methods are lattice-theoretic, and the only undefined relations are incidence relations. Three equivalent forms of the "fundamental axiom" are considered (analogous, in projective geometry, to the various equivalent statements of the fundamental theorem). Co-ordinates and the equations of circles and spheres are introduced synthetically, and the associated number field of the geometry is shown to be an ordered field F in which every positive number is a square. Contact is made with work of Gorn, Izumi, Pieri, and van der Waerden and Smid. (Received November 15, 1948.)

193*t*. M. O. Reade: *Generalizations to space of a theorem of Fédoroff.*

Fédoroff has shown that if $f(z)$ is continuous in a bounded, simply-connected domain $\mathfrak{D}$, then a necessary and sufficient condition that $f(z)$ be analytic in $\mathfrak{D}$ is that $\int\int(z-z_0)f(z)dxdy=0$, the integration being over any disc $D(z_0, r)$, with center z_0 and radius r in $\mathfrak{D}$ (Rec. Math. (Mat. Sbornik) vol 41 (1934) p. 92). Generalizations of the following type are proved in this paper. Let $U_j(z)\equiv U_j(x, y), j=1, 2, 3$, be real-valued and have continuous partial derivatives of the third order in $\mathfrak{D}$. Then a necessary and sufficient condition that the functions $U_j(z)$ either (1) map $\mathfrak{D}$ isothermically on a surface that lies on a sphere of finite non-null radius, such that circles are mapped on circles, or (2) be the coordinate functions of a minimal surface in isothermic representation, is that $\sum_{j=1}^{3} [\iint (z-z_0) U_j(z)dxdy]^2=0$, the integration being over any disc $D(z_0, r)$ in $\mathcal{D}$. This result is analogous to a result obtained by Reade and Beckenbach (Trans. Amer. Math. Soc. vol. 49 (1941) p. 354). (Received November 16, 1948.)

194. T. G. Room: *Arithmetic of curves on certain surfaces.*

The transversal lines to a line, l, lying on a cubic surface, Π, and a line, v, not on Π, determine an involutory self-transformation $\mathcal{L}$ of Π. The curve $\mathcal{L}l$, together with v, determines a new involutory transformation of Π, and the 27 lines on Π determine an enumerable family of transformations. These form a groupoid in which the product of any three members is a member. If curves $c_1, \cdots, c_8$ are taken as a base on Π, then under $\mathcal{L}$ the curves c become curves $\mathcal{L}c$, expressible as Lc, where L is an involutory unitary matrix of integers. The matrices L may be expressed both as products formed from 8 basic matrices, and as sums of integer multiples of these 8, the integers being quadratic functions of 7 parameters. The matrices L may be interpreted as involutory collineations in [7], in which the base elements are a family of lines through a point W, and their polar [5]'s with regard to a quadric through W, and may thence be represented by an enumerable system of points in [6]. The figure formed by these points is an expansion of the figure of two perspective simplexes in [5] by which the 27 lines are ordinarily represented. (Received November 22, 1948.)

195*t*. W. R. Utz: *Almost periodic geodesics on manifolds of hyperbolic type.*

The manifolds considered are secured by the identification of congruent points (under a Fuchsian group) of certain Riemannian manifolds whose fundamental forms are defined on the interior of the unit sphere of Euclidean n-space. The principal theorem shows that almost periodic, nonperiodic geodesics exist on these manifolds when such geodesics exist on a companion manifold of constant negative curvature. In the case $n=2$, the almost periodic, nonperiodic geodesics on a class of manifolds of constant negative curvature are shown to be everywhere dense when the group employed is of the first kind. The latter result is an improvement on an unpublished result due to Marston Morse. (Received November 15, 1948.)

Logic and Foundations

196. Henry Blumberg: *Conception of set; elimination of the paradoxes of set theory.*

This paper sets forth a conception of set—termed "genetic"—which, as the author submits, constitutes an appropriate solution of the problem of the set-theoretic paradoxes. This conception is close to mathematical experience, requires no special devices or new constructions, and permits a logically unimpeachable development of the salient points of Cantor's discoveries—in particular, of Zermelo's theorem on the normal order. The principal idea is to validate the concepts and modes of conceptual derivation which mathematicians had no hesitation in accrediting before the phenomenon of the paradoxes, and developing the implications of such validation. This ideas is supported by the fact that the set-theoretic paradoxes may be eliminated frontally; in other words, as the author shows, the paradox maker, in every case, commits an error in the argument allegedly establishing the paradox. Such direct refutation is facilitated by the proposed genetic conception of set. This conception does not fix once for all what a set is; a satisfactory conception of set cannot be expected to do this. Legitimate sets are derived from accredited sets by accredited associations; but the phrases "all legitimate sets" and "there exists a legitimate set" have no meaning. There is clarification, too, of the deficiencies of other proposed conceptions of set. (Received November 12, 1948.)

197*t*. H. B. Curry: *A theory of formal deducibility.*

This is a revision of a paper read in 1937 (Bull. Amer Math. Soc. Abstract 43-9-325). The fundamental idea is to apply the inferential-rule methods of Gentzen to propositions relating to a formal system, so as to make a semantic analysis of the "compound" propositions formed from the elementary propositions of the system by the connectives of propositional algebra and predicate calculus. The new results relate chiefly to extending these methods to include the classical as well as the intuitionistic approaches. When negation is introduced four types of system are considered, namely: M, the minimal system (Johansson); J, the intuitionistic system (Heyting); K, the classical system; and D, a minimal system with excluded middle (Johansson), applicable when the underlying formal system is decidable. For each type of system there are three types of formulation called respectively L, T, H; the L formulation is like that so-called by Gentzen, T is Gentzen's "natural" formulation, and H is a more orthodox calculus! Relations between these types of system and formulation are considered, including a generalization of the Glivenko theorem. The paper will be published in booklet form by the University of Notre Dame. (Received November 16, 1948.)

198. H. B. Curry: *The elimination theorem when necessity is present.*

In the Notre Dame lectures on formal deducibility (see the preceding abstract) a proof of the fundamental theorem, called the elimination theorem (Genzen's "Hauptsatz"), was lacking for the case of systems involving necessity. This hiatus is filled in the present paper. This enables the treatment of modal systems to be completed; and we now have relations between L, T, and H, formulations of propositional algebra just like those for non-modal systems. In particular the procedure gives a decision process for the Lewis system S4; however the relation of this procedure to those previously known for this system has not been investigated. The generalized form of the elimination theorem so obtained leads to some simplifications in the previous theory, notably the connections between the LC (classical positive) and LA (intuitionistic positive) systems. (Received November 16, 1948.)

199*t*. H. B. Curry: *The permutability of rules in the classical inferential calculus.*

Suppose we have a rule theoretic system à la Gentzen with elementary statements of the form $X_1, X_2, \cdots, X_m \rightarrow Y_1, Y_2, \cdots, Y_n$. With reference to the rules of such a system we distinguish as parameters those X_i, Y_j which go over unchanged from premises to conclusion, as components those which appear in the premises but not in the conclusion, and as principal constituents those which occur in the conclusion only. Suppose the rules are such that the same parameters appear in all the premises, and that parameters can be added to and deleted from all premises and conclusion simultaneously without destroying the validity of the inference. Then the following theorem is true: if a rule R_1 is followed by a rule R_2 in such a way that the principal constituents for R_1 are parameters for R_2, then the rules can be interchanged. This simple observation includes the strong form of the elimination theorem ("Hauptsatz") which is valid in Gentzen's system LK. In the system LJ it is not possible to add parameters on the right, and consequently a proof of the elimination theorem by this method breaks down. (Received November 15, 1948.)

200. Marshall Hall: *The decision problem for semigroups with two generators.*

Emil Post has shown (Journal of Symbolic Logic vol. 12 (1947) pp. 1–11) that the decision problem for semigroups is unsolvable. It is shown here that the decision problem for an arbitrary semigroup may be reduced to that for an appropriate semigroup with two generators. Hence it follows that the decision problem is unsolvable for semigroups with two generators. (Received December 14, 1948.)

201. Ilse L. Novak: *The relative consistency of von Neumann's and Zermelo's axioms for set theory.*

The system of axioms for class and set theory which was adapted from von Neumann's by Bernays (Journal of Symbolic Logic vol. 2 (1937) p. 65) and Gödel (*Consistency of the continuum hypothesis*, Princeton, 1940) differs from Zermelo's (including the Aussonderungs- and Ersetzungs-axiome) by admitting classes (non-elements) as well as sets. In the present paper a model of the von Neumann-Bernays-Gödel system is constructed in the syntax of Zermelo's. The syntax employed has axioms stating (i) certain basic signs and their combinations exist, (ii) quantification theory, (iii) induction on the length of formulas and (iv) identity theory may be used to derive metatheorems, (v) Zermelo's system is consistent. The "ϵ" of Zermelo's system is reinterpreted as a syntactic relation between names of sets with help of a syntactically defined predicate which proves true of all Zermelo's theorems and true of any two formulas if and only if true of their conjunction, and true of any formula if and only if not true of its denial. The construction turns on the fact that the well-ordering hypothesis is consistent with Zermelo's system. The existence of this model shows von Neumann's system consistent relative to this syntax of Zermelo's system. (Received November 13, 1948.)

202. Ernst Snapper and M. A. Zorn: *On transfinite induction.*

A new mathematical proof for a variant of transfinite induction up to the first epsilon-number is given. The proof is (i) capable of generalization, (ii) of metamathematical interest. (Received November 16, 1948.)

Statistics and Probability

203*t*. D. A. Darling: *Note on a distribution.*

If $X_1, X_2, \cdots, X_n$ are positive independent identically distributed random variables with the common continuous distribution $F(x)$ and if $X^* = \max(X_i)$ we define $Z_n = (X_1+X_2+\cdots+X_n)/X^*$. If $\phi(x)$ is the density of X_i, the characteristic function of Z_n is $E(\exp(itZ_n)) = n(\exp(it))\int_0^\infty z^{n-1}\phi(z)(\int_0^1 \exp(ity)\phi(yz)dy)^{n-1}dz$. The cases of practical importance where the X's are distributed as χ^2 with m degrees of freedom or as the range of m independent variables appear difficult to calculate explicitly. Certain limiting distributions are considered. (Received November 15, 1948.)

Topology

204. R. D. Anderson: *A characterization of a certain class of subcontinua of the sphere.*

Let M be a subcontinuum of a two-dimensional sphere Σ. The author shows that in order that there exist an upper semicontinuous collection G of mutually exclusive continua filling up M such that G with respect to its elements as points is topologically equivalent to Σ, it is necessary and sufficient that there exist two uncountable collections E and F of mutually exclusive subcontinua of M such that each element of E intersects each element of F. Other results of a similar nature are obtained by the author. The above theorem can be applied to certain sums of two pseudo arcs in the plane. (Received November 15, 1948.)

205t. R. D. Anderson: *Concerning upper semicontinuous collections of continua in the sphere.*

A collection F of continua will be said to have the W property provided that if H_1 is any element of F and x is any point of F^*-H_1 there exists a continuum H_2 of F containing x and lying in F^*-H_1 and if y is any point of $F^*-H_1-H_2$ there exists a continuum H_3 of F containing y and lying in $F^*-H_1-H_2$ and if z is any point of $F^*-H_1-H_2-H_3$ there exists a continuum H_4 of F containing z and lying in $F^*-H_1-H_2-H_3$ and there exists a set E of six continua of F each containing exactly two of the continua H_1, H_2, H_3, H_4 such that the common part of any two continua of E if it exists is one continuum of the set H_1, H_2, H_3 and H_4. Let M be a subcontinuum of a two-dimensional sphere Σ. The author shows that in order that there exist an upper semicontinuous collection G of mutually exclusive continua filling up M such that G with respect to its elements as points is topologically equivalent to Σ it is necessary and sufficient that there exist a nondegenerate collection F of subcontinua of M covering M having the W property. A V property is also defined with similar application. (Received November 15, 1948.)

206. S. S. Cairns: *An elementary proof of the Jordan curve theorem.*

An elementary and relatively brief proof is given that a simple closed curve in the plane determines two regions, one of which is a 2-cell. The argument is better adapted for presentation to a beginning graduate topology class than are other proofs known to the author. (Received November 9, 1948.)

207. R. P. Dilworth: *The space of normal functions.*

If f is a bounded real function on a compact Hausdorff space S, let f^* and f_* denote the upper and lower limit functions of f respectively. If $(f^*)_*=f_*$ and $(f_*)^*=f^*$, then f is said to be ***normal.*** Let $N(S)$ denote the set of all normal functions on S. $N(S)$ is a complete lattice under the partial ordering $f^*\geqq g_*$. The following results are obtained: (1) $N(S)$ is lattice isomorphic to the lattice of all real continuous functions on a compact Hausdorff space T_S; (2) T_S is extremally disconnected; (3) S is homeomorphic to a factor space of T_S; (4) T_S is essentially the minimal compact Housdorff space having properties (2) and (3). (Received November 18, 1948.)

208. C. H. Dowker: *Čech cohomology groups and the axioms.*

A definition is given for the Čech cohomology groups $H^n(X, A)$ of a topological space X relative to a not necessarily closed subset A. Infinite coverings by open sets are used. With this definition it is shown that the Čech cohomology theory satisfies the Eilenberg-Steenrod axioms, including the homotopy axiom, for arbitrary pairs (X, A). (Received November 12, 1948.)

209*t*. Marianne R. Freundlich: *Banach spaces and group duality.*

It is shown that the apparent similarity of the Pontrjagin duality theorem for locally compact groups to the duality existing in reflexive Banach spaces is more than superficial, in the following sense: for reflexive Banach spaces considered as topological groups the Pontrjagin theorem holds verbatim; that is, the character group of the character group of a reflexive Banach space B is topologically and algebraically isomorphic with B. The standard topologies are assigned to character groups. The proof makes use of results of Arens (*Duality in linear spaces*, Duke Math. J. vol. 14 (1947) pp. 784–794) and Myers (*Equicontinuous sets of mappings*, Ann. of Math. vol. 47 (1946) pp. 496–502). (Received November 15, 1948.)

210*t*. R. E. Fullerton: *On a semi-group of subsets of a linear space.*

A semi-group of subsets of a space is defined to be a family $\mathcal{J}$ of subsets such that if S_1 and S_2 are in $\mathcal{J}$, then $S_1 \cap S_2 \in F$. It is proved that if S is a subset of a linear space, closed in a certain weak sense and possessing an extreme point, then all translates of S form a semi-group only if S is a convex cone. (Received November 16, 1948.)

211. V. G. Gorciu: *Interior transformations in the case of uniform, non-metrizable spaces.*

Two uniform, non-metrizable compact spaces, A and B, are defined as neighborhood-spaces by assigning an indexed set of neighborhoods to every point. The set I of indices is directed, non-denumerable and has no last element. The following definitions are introduced: A property P, depending on an index α, is "of non-cofinal character" if and only if the set of indices satisfying that property is a non-cofinal subset of I. A point x is a "limit point" of a directed set $\{x_\alpha\}$ if and only if every neighborhood of x contains a cofinal subset of $\{x_\alpha\}$. Denote by $P(\alpha)$ the following property: For any β there exists an α such that for any $y \in B$ and any $x \in T^{-1}(y)$, $T[V_\beta(x)] \supset V_\alpha(y)$. The following theorem can now be stated: In order that the continuous mapping $T(A) = B$ be an interior transformation, it is necessary and sufficient that $\sim P$ be of non-cofinal character. This theorem can be regarded as an extension of a result of G. T. Whyburn. (Received December 9, 1948.)

212*t*. J. W. Green and William Gustin: *On the direct sum of continua.*

Let X be a real vector space of finite dimension n; and let a_ν (ν denotes a variable index ranging over the n indices $1, \cdots, n$) be n linearly independent vectors in X. Consider n continua Q_ν in X, the continuum Q_ν passing through the origin and through a_ν. It is shown that every point x in the space X is of the form $\sum(\alpha_\nu a_\nu + q_\nu)$ where q_ν lies in Q_ν and the α_ν are integers. (Received November 16, 1948.)

213. O. G. Harrold: *Euclidean domains with uniformly abelian local fundamental groups.*

Let A be an arcwise connected subset of a topological space X. A group $C(A, X)$

is defined as follows. Closed paths in A are considered based at $p \in A$ that are homotopic in X to a commutator of paths in X. Classes of paths and multiplication of classes are defined as for the fundamental group. A local analogue of the fact that the commutator-quotient group of the fundamental group is the 1-dimensional homology group over the integers is given. If for each $p \in X$ and each neighborhood U of p rel. A there is a neighborhood V of p rel. A such that for each component V_1 of V, $C(V_1, U) = 0$, A is said to have uniformly abelian local fundamental groups. Included in the results are the following. Let X be spherical n-space and A the complement of a closed, topological k-cell, $k = 1, 2, \cdots, n$. If A has the above defined property, $\pi_1(A)$ is trivial. If A is the complement of a closed, totally disconnected subset and A has the above property, $\pi_1(A)$ is trivial. (Received November 15, 1948.)

214. Edwin Hewitt: *Functionals related to normal spaces.*

Results of F. Riesz, Kakutani, and Markov are extended to obtain the following theorem. Let X be any normal space and let $C(X, R)$ be the space of all continuous real-valued functions defined on X. Let ϕ be any real-valued function defined on $C(X, R)$ such that (1) $\phi(f+g) = \phi(f) + \phi(g)$; (2) $\phi(\alpha f) = \alpha\phi(f)$, where α is any real number; (3) $f \geqq 0$ implies $\phi(f) \geqq 0$. Then there exists a measure σ on X such that all Borel sets are measurable and such that $\phi(f) = \int_X f d\sigma$. If X admits unbounded continuous real-valued functions, then the measure σ is subject to certain strong restrictions, which are described. (Received November 12, 1948.)

215*t*. E. J. McShane: *Images of sets satisfying the conditions of Baire.*

Assume $S \subset X_1$, a topological space; F a family of functions f with open domain D_f containing S and mapping open subsets of D_f on open subsets of X_2, a topological space. If S is of second category and satisfies the condition of Baire, and for each $f_0 \in F$ and each $x_0 \in D_f$ at which S is of second category the aggregate of all inverse images of $f_0(x_0)$ under all $f \in F$ is of second category at x_0, then there is a set S_0 consisting of all of S except a set of first category such that the union of the $f(S_0)$ for $f \in F$ is open. The corollaries include and extend a number of known theorems on subgroups and semi-groups of topological groups, on additive mappings and on "mid-point-convex" sets and functions. (Received November 19, 1948.)

216. W. S. Massey: *Classification of mappings of an $(n+1)$-dimensional space into an n-sphere.*

The problem of determining all homotopy classes of continuous maps of an $(n+1)$-dimensional complex K into an n-sphere S^n was solved by Steenrod (Ann. of Math. vol. 48 (1947) pp. 290–320). This solution made essential use of the fact that the homotopy group $\pi_{n+1}(S^n)$ is cyclic of order two, a result due to Freudenthal and Pontrjagin. In this paper another proof is given of this homotopy classification theorem which does not require a knowledge of $\pi_{n+1}(S^n)$. The fact that $\pi_{n+1}(S^n)$ is cyclic of order two then follows as a corollary. The proof is based on an extension theorem of the following kind. Let X be a compact metric space of dimension $\leqq n+2$, let A be a closed subset of X, and $f: A \rightarrow S^n$ a continuous map. New operations are introduced which enable one to state necessary and sufficient conditions for the existence of a continuous map $F: X \rightarrow S^n$ which is an extension of f. In case (X, A) is a simplicial pair, and f is a simplicial map, these new operations are effectively calculable. The extension theorem is proved by an inductive procedure similar to that used in the proof of the Hopf Extension Theorem by Hurewicz and Wallman (*Dimension theory*, p. 142). (Received November 13, 1948.)

217. G. E. Schweigert: *Basic invariance theorems for onto-transformations.*

It is well known that, for homeomorphisms, the closure (more generally P-hull), complement, union, intersection and certain limits of invariant sets are themselves invariant. These are investigated in the setting $f(M)=M$ (where $f^{-1}(x)$ is not necessarily a point); the theorems show a wide divergence in the assumptions needed to reach the desired conclusions. Some sample theorems: The P-hull of an invariant set is invariant provided f and f^{-1} preserve property P; $A\subset f(A)$ implies the union of $f^n(A)$ is invariant; f continuous, B closed in a compact Hausdorff space and $f(B)\subset B$ implies $f(\Pi)=\Pi$ where Π is the intersection; $K=\bigcap K_\alpha$, K_α invariant, implies $f(K)\subset K$, but $K\subset f(K)$ does not follow when f is interior. If L and l are limits superior and inferior for collections of invariant sets, then $f(L)=L$ and $f(l)\subset l$ follow the usual assumptions, but $l\subset f(l)$ does not. The corresponding statements for strong invariance, namely $f^{-1}(A)=A$, show better resemblance to the homeomorphism case. The results are, in general, rich enough to invite an interesting "orbit-theory." (Received November 6, 1948.)

218. G. T. Whyburn: *Expansive mappings.*

Let A and B be locally connected generalized continua and let f be a mapping ($=$continuous transformation) of A into B. Then f is *expansive* provided A is the union of a strictly monotone increasing sequence $[R_n]$ of conditionally compact regions, that is, $\bar{R}_n\subset R_{n+1}$, such that if $F_n=f[F(R_n)]$ where $F(R_n)$ denotes the boundary of R_n, any compact set in B intersects at most a finite number of the sets F_n. Also f is *strongly interior* or open provided the image of every open set in A is open in B. A non-constant entire analytic function (which is always strongly interior) is expansive provided that for some sequence of circles $|z_n|=r_n$ with $r_n\rightarrow\infty$ we have $\min_{|z|=r_n}|f(z_n)|\rightarrow\infty$ and a converse holds if we use simple closed curves. Non-constant entire functions of order less than 1/2 are thus expansive. If f is strongly interior and expansive, it maps A *onto* B; and, indeed, either for each $z\in B$, $f^{-1}(z)$ is compact and nonempty or else for each $z\in B$, $f^{-1}(z)$ is non-compact and thus infinite. A mapping of A onto B is expansive if and only if each component of the inverse of a continuum in B is compact. (Received October 1, 1948.)

219*t*. G. T. Whyburn: *Quasi-interior mappings.*

A mapping f of one locally connected generalized continuum A into another one B is *quasi-interior* provided that for any $y\in f(A)$ and any open set U in A containing a compact component of $f^{-1}(y)$, y is interior rel. B to $f(U)$. A mapping is quasi-interior if and only if for each region R in B, each conditionally compact non-empty component of $f^{-1}(R)$ maps onto R under f. There exist quasi-interior mappings which preserve neither local compactness nor local connectedness. If the sequence of quasi-interior mappings of A into B converges to the mapping f on A, the convergence being uniform on each compact set in A, then f is quasi-interior. Product and factor theorems are also established. Clearly every strongly interior mapping is quasi-interior and every light quasi-interior mapping is strongly interior. Also, in case A is compact as well as locally connected and connected, the property of quasi-interiority of a mapping turns out to be the same as quasi-monotoneity as defined originally by A. D. Wallace. (Received October 1, 1948.)

220t. G. T. Whyburn: *On compact mappings.*

A study was made of mappings $f(A)=B$ (A and B separable metric) having the property that the inverse of any compact set in B is compact. This property is equivalent to the mapping being closed and having compact point-inverses. Upon completion we learned that Vainstein had recently studied mappings having the latter property, calling them *compact* mappings. Thus we adopt his term and report some of our results that do not seem to duplicate his. If $f(A)=B$ is compact, for any $y \in B$ and any open set U in A containing $f^{-1}(y)$, y is interior to $f(U)$. Thus local compactness and local connectedness are invariant under compact mappings. If f is compact and monotone, the inverse of every connected set is connected; thus f is *strongly monotone,* that is, the inverse of a continuum is a continuum. If A is a locally connected generalized continuum, for a monotone mapping on A, the properties of compactness, strong monotoneity and quasi-interiority are equivalent. Compact mappings have the group property and however a compact mapping be factored continuously: $f=f_2f_1$, *both* factors are necessarily compact. Any two monotone light factorizations of a compact mapping are topologically equivalent. Such factorizations always exist (as remarked also by Vainstein). Other product and factor theorems are obtained. (Received October 1, 1948.)

J. W. T. Youngs,
Associate Secretary

APPENDIX

Excerpts from Report of Treasurer

December 22, 1948

To the Board of Trustees of the
American Mathematical Society

Gentlemen:

I have the honor to submit herewith the report of the Treasurer for the fiscal year ended November 30, 1948, with certain pertinent comments.

Investment Portfolio

On November 30, 1948, the market value of securities held for Invested Funds exceeded book value by $2,789, but the market value of securities held for Current Funds was less than book value by $1,626. On the whole portfolio, the market value therefore exceeds book value by $1,163. This showing is less favorable than in recent years because security prices have declined considerably in the last quarter of the fiscal year. Nevertheless, reserves held in accounts "Reserve for Investment Losses" ($4,386) and "Profit on Sales of Securities" ($18,174) may still be considered adequate protection against contingent depreciation in market value.

The following is a summary of the changes in security holdings made during the year.

Acquired	
$5,000	U. S. Treasury 2s 1954–52
75 shares	Consumers Power Co. com. pfd. $4.52
100 shares	Continental Oil Co. com.
3 shares	Texas Co. com.
200 shares	Union Carbide and Carbon Corp. cap.
1 share	Standard Oil Co. of New Jersey cap.
Sold	
$5,000	U. S. Treasury 2 1/2s 1972–67
100 shares	Union Carbide and Carbon Corp. cap.
200 shares	Consolidated Edison Co. of New York Inc. com.
20 shares	Texas Co. com.
50 shares	Timken Roller Bearing Co. cap.
1 share	Standard Oil Co. of New Jersey cap.

The investment portfolio, valued at market November 30, 1948, now includes Government bonds 36.3 per cent, other bonds 8.1 per cent, preferred stock 15.4 per cent, common stock 35.0 per cent, cash in savings banks 5.2 per cent.

Income from Investments

Income received during the year from investment of Current Funds amounted to $2,717. This represents a return of 3.0 per cent computed on average book value of investments. Income on Invested Funds amounted to $8,437, representing a return of 4.3 per cent. Total investment income from all sources was $11,154, representing a return over 3.9 per cent. These rates of return are higher than in 1947.

Income from the Henderson Estate was $4,990; in 1947 it was $4,850.

Changes in Net Assets

Net assets increased by $3,756 during the year. That this showing is somewhat better than had been anticipated is to be attributed principally to record-breaking sales of publications, to receipts from dues and initiation fees amounting to $3,000 more than estimates, and to increased yield from investments. Included in receipts is the sum of $1,000 from the estate of J. K. Whittemore. Navy Contract N7 onr 429 provided $22,500 to cover editorial expenses of Mathematical Reviews for June 20, 1947 through October 20, 1948. About $7,000 more will be received under this contract before it expires in March, 1949. Without this item, net assets would have shown a decrease of nearly $19,000.

Included in assets is the sum of $480 due the Society under Navy Contract N8 onr 553 for translation of papers from the Russian and other unfamiliar languages. It has seemed advisable thus to treat expenses paid by the Society and billed under this contract, although it is not our present practice to list other accounts receivable as assets nor accounts payable as liabilities.

During the year the account for International Congress was restored to active status by direction of the Board.

Decrease in Surplus

Surplus account shows a decrease of $5,349, somewhat less than in 1947. During the year it was necessary to transfer $10,000 from Surplus to Colloquium account in order to pay for the heavy volume of printing and reprinting currently under way.

It should be noted also that Surplus account reflects the dues and initiation fee income resulting from continued rapid growth of membership, as well as the additional income resulting from the new dues rates that became effective on January 1, 1948.

Increase in Inventory

In interpreting the report and the preceding remarks, cognizance should be taken of the unusual situation with regard to inventory. Because change in inventory tends in normal years to be relatively small and quite steady, it has not been the practice of the Society to include an inventory account in financial statements. On December 1, 1947, stocks of books were unusually low. During the year, printing expenses for Colloquium account amounted to $25,500. For 1941–1947 inclusive they totalled only about $14,000. A study has therefore been made of the change in inventory of books, not including periodicals, resulting from the year's activities. It appears that, valuing inventory at prices to members and allowing for handling charges, there was an increase in inventory from about $11,000 at the beginning of the year to about $37,000 at the end. No corresponding study was made for periodicals, but it is probable that there was some increase here also. This increase to a considerable extent balances the above mentioned somewhat disturbing change in net assets. For, while to some extent the financial soundness of the Society depends on the time required to reconvert inventory into cash, the current volume of sales is, as already noted, very high, so that there is no immediate cause for anxiety in the increased inventory.

At the same time it is necessary to be watchful that suitable sources of income are obtained to cover the steadily rising costs for the Bulletin, Transactions, and Mathematical Reviews, and for salaries.

In concluding this last letter of transmission which I shall have the privilege of writing, it may be helpful to give a brief indication of the tremendous growth of the Society during the past eleven years as reflected in its finances by citing side by side, without comment, a few figures from the Treasurer's reports for 1937 and for 1948.

	1937	1948
Income		
Dues and Initiation Fees—Individuals	$14,500	$31,700
Dues—Institutions	6,400	9,600
Investment Income	4,300	16,100
. . .	. . .	. . .
Total Income Excluding Income from Sales of Publications	$26,800	$ 84,200
Sales of Publications	9,700	51,400
Total Income	$36,500	$135,600

Expenses		
General Expenses—Offices, Officers, Salaries, Etc.	$ 14,400	$ 54,300
Printing	14,200	75,200
. . .	. . .	. . .
Total Expenses	$ 31,200	$132,000
Total Assets (Not Including Inventories)	$130,800	$286,900

Finally, may I be permitted to express here the pleasure and satisfaction which have accompanied the work of the treasurership during my rather long tenure, and which have resulted from the unfailing and cordial cooperation of the other officers and of the employees of the Society. Especially has the work of Miss Evelyn M. Hull as Office Manager been indispensable, for without her excellent judgment and thorough familiarity with all the details of the finances, the difficulties of the treasurer would have been nearly insuperable.

And to the members of the Board, past and present, I offer sincere gratitude not only for the joy of working together but for their repeated expressions of personal appreciation.

Respectfully submitted,
BENNINGTON P. GILL,
Treasurer

BALANCE SHEET

	November 30, 1948	November 30, 1947
Assets		
CURRENT FUNDS:		
Cash	$ 16,292.01	$ 16,315.50
Account Receivable from United States Government	480.21	
Investments	77,500.19	70,559.50
	$ 94,272.41	$ 86,875.00
INVESTED FUNDS:		
Cash	$ 531.01	$ 1,287.90
Investments	192,102.21	194,987.15
	$192,633.22	$196,275.05
TOTAL ASSETS	$286,905.63	$283,150.05
Liabilities		
CURRENT FUNDS:		
Mathematical Reviews	$ 13,992.32	$ 1,897.50
Colloquium	10,813.01	11,708.12
Mathematical Surveys	3,787.21	3,734.70
Symposia on Applied Mathematics	2,595.25	2,700.00
Birkhoff Memorial Project	3,131.68	3,061.68
International Congress	5,484.84	6,413.49
Committee on Aid to Devastated Libraries	128.96	1,320.50
Policy Committee	97.64	
Prize Funds and Other Special Funds Accumulated Income	8,238.39	6,848.12
Reprinting Funds	6,847.71	6,165.74
Sinking Fund	1,156.05	1,044.54
Profit on Sales of Securities	2,032.98	982.60
Miscellaneous	617.50	300.45
Surplus	35,348.87	40,697.56
	$ 94,272.41	$ 86,875.00
INVESTED FUNDS:		
Endowment Fund Principal	$ 71,000.00	$ 71,000.00
Prize Funds and Other Special Funds	33,033.22	32,033.22
Life Membership and Subscription Reserve	3,073.09	3,407.61
Mathematical Reviews	65,000.00	65,000.00
Colloquium		5,000.00
Reserve for Investment Losses	4,385.89	4,385.89
Profit on Sales of Securities	16,141.02	15,448.33
	$192,633.22	$196,275.05
TOTAL LIABILITIES	$286,905.63	$283,150.05

SUMMARY STATEMENT OF INCOME AND EXPENDITURES
1947–1948

	1948		1947	
	Receipts	Disbursements	Receipts	Disbursements
General Receipts:				
Dues—Ordinary Memberships	$29,120		$21,628	
Dues—Contributing Memberships	712		932	
Dues—Institutional Memberships	9,582		8,012	
Initiation Fees	1,880		1,195	
Investment Income	11,953		11,056	
Miscellaneous	191		288	
General Disbursements:				
Secretaries		$ 13,395		$11,301
Treasurer		1,182		1,470
Librarian		2,132		1,478
Committee Expense		831		
Editorial Office Expense		736		305
Office Remodeling		55		
Furniture and Fixtures		750		609
Miscellaneous		103		65
Total	$53,438	$19,184	$43,111	$15,228
Excess of General Receipts	$34,254		$27,883	
Publications:				
Bulletin	$ 4,318	$ 23,950	$ 3,180	$21,940
Transactions	8,647	15,490	7,398	13,065
Bulletin and Transactions—Back Volumes	6,156	5,474	2,363	1,978
Mathematical Reviews	46,641	34,546	19,818	25,310
Colloquium Publications	22,471	28,366	8,135	5,923
Mathematical Surveys	263	211	2,489	551
Semicentennial Publications	114	2	93	9
Proceedings Symposia on Applied Mathematics		105	2,700	
Birkhoff Papers	70		3,225	164
Annals of Mathematics				1,000
American Journal				2,250
Total	$88,680	$110,144	$49,401	$72,190
Excess Cost of Publications		$ 21,464		$22,789
Other:				
Policy Committee for Mathematics	$ 675	$ 577	$ 1,500	$ 2,45$^{2}_{4}$
International Congress		929	95	
Prize and Other Special Funds	2,390		2,257	
Committee on Aid to Devastated Libraries	20	1,212	2,068	748
Profit on Sales of Securities	1,743		1,904	
Miscellaneous	317		141	
Appropriations:				
Policy Committee	39	500		
Mathematical Reviews		1,000		1,000
Colloquium		10,000		
Symposia on Applied Mathematics				2,700
Committee on Aid to Devastated Libraries				1,500
Mathematical Surveys				2,000
Total	$ 5,184	$ 14,218	$ 7,965	$10,404
Difference		$ 9,034		$ 2,439
Net Change in Assets	$ 3,756		$ 2,655	
Assets Beginning of Year	$283,150		$280,495	
Assets End of Year	$286,906		$283,150	