THE NOVEMBER MEETING IN CHICAGO

The four hundred fortieth meeting of the American Mathematical Society was held at the Museum of Science and Industry, Chicago, Illinois, on Friday and Saturday, November 26–27, 1948. The total attendance was about 200 including the following 136 members of the Society:

A. A. Albert, W. R. Allen, R. V. Andree, M. L. Anthony, Max Astrachan, W. L. Ayres, Reinhold Baer, Walter Bartky, H. R. Beveridge, S. F. Bibb, R. H. Bing, K. E. Bisshopp, W. M. Boothby, A. J. Brandt, J. W. Butler, K. H. Carlson, R. E. Carr, Harold Chatland, Herman Chernoff, E. W. Chittenden, W. G. Clark, E. H. Clarke, H. E. Clarkson, E. G. H. Comfort, J. J. Corliss, A. R. Craw, H. J. Curtis, G. K. DelFranco, Flora Dinkines, J. L. Doob, John Dyer-Bennet, B. J. Eisenstadt, H. M. Esser, H. S. Everett, Chester Feldman, J. V. Finch, C. H. Fischer, Harley Flanders, L. R. Ford, R. S. Fouch, Evelyn Frank, C. G. Fry, R. E. Fullerton, S. H. Gould, L. M. Graves, D. E. Green, L. J. Green, L. W. Griffiths, M. M. Gutterman, P. R. Halmos, Frank Harary, E. D. Hellinger, Vaclav Hlavaty, G. P. Hochschild, Carl Holtom, L. A. Hostinsky, S. P. Hughart, H. K. Hughes, Ralph Hull, Meyer Jerison, Louis Joseph, Samuel Kaplan, William Karush, Leo Katz, Fulton Koehler, J. C. Koken, W. C. Krathwohl, M. Z. Krzywoblocki, E. P. Lane, J. S. Leech, Saunders MacLane, Kenneth May, D. M. Merriell, Clair E. Miller, D. D. Miller, J. M. Mitchell, C. W. Moran, E. J. Moulton, Leopoldo Nachbin, E. D. Nering, August Newlander, O. M. Nikodym, E. P. Northrop, F. S. Nowlan, J. M. H. Olmsted, Daniel Orloff, E. H. Ostrow, Gordon Pall, Sam Perlis, D. H. Potts, D. W. Pounder, G. B. Price, O. W. Rechard, H. W. Reddick, W. P. Reid, W. T. Reid, Haim Reingold, P. R. Rider, L. A. Ringenberg, V. N. Robinson, Arthur Rosenthal, A. E. Ross, Herman Rubin, J. M. Sachs, Hans Samuelson, R. G. Sanger, L. J. Savage, A. C. Schaeffer, O. F. G. Schilling, Lowell Schoenfeld, W. T. Scott, A. S. Shapiro, Harold Shniad, D. N. Silver, G. F. Simmons, H. A. Simmons, Annette Sinclair, D. M. Smiley, M. F. Smiley, G. W. Smith, W. S. Snyder, E. H. Spanier, D. C. Spencer, D. M. Stone, A. F. Svoboda, L. W. Swanson, T. T. Tanimoto, A. H. Taub, E. F. Trombley, H. E. Vaughan, G. L. Walker, S. E. Walkley, D. R. Waterman, André Weil, J. W. T. Youngs, Antoni Zygmund.

The Committee to Select Hour Speakers for Western Sectional Meetings had invited two speakers for the occasion. Professor P. R. Halmos, of the University of Chicago, spoke on *Measurable transformations* at 2:00 P.M., Friday; Professor Saunders MacLane, of the University of Chicago, on *Duality for groups* at 10:00 A.M., Saturday. Presiding officers at these lectures were Professors J. L. Doob and Reinhold Baer, respectively.

Two sessions were held for contributed papers: Friday, 10:30 A.M., Professor Arthur Rosenthal presiding; and Friday, 3:15 P.M., Professor E. W. Chittenden presiding.

At the session on Saturday morning, Professor L. M. Graves moved that the Associate Secretary be instructed to write a letter of thanks to Major Lenox Lohr, Director of the Museum of Science and Industry, for the hospitality which the Museum had extended to the Society. The motion was seconded and passed unanimously. The Associate Secretary has carried out these instructions.

Social life at the meeting was enlivened by a party held by Mrs. Dorothy MacLane for visiting mathematicians on Friday evening from 4:30 to 7:00 P.M.

Abstracts of all papers presented at the meeting are given below. Papers read by title are indicated by the letter "*t*." Paper number 48 was presented by Professor Chatland.

ALGEBRA AND THEORY OF NUMBERS

48. Harold Chatland and Harold Davenport: *Euclid's algorithm in real quadratic fields.*

The question of the existence of Euclid's algorithm in real quadratic fields, $k(m^{1/2})$ has been settled except for m a prime, of the form $24n+1$. For $m>128^2$ Davenport proved the nonexistence of the algorithm. Chatland then showed that except for $m=193$, 241, 313, 337, 457, 601 the algorithm is invalid for m a prime of the form $24n+1$. In this paper it is proved that the algorithm does not exist in any of these six cases. Hence, as a result of all investigations it may be said that Euclid's algorithm exists in the real field $k(m^{1/2})$ if and only if m has one of the values 2, 3, 5, 6, 7, 11, 13, 17, 19, 21, 29, 33, 37, 41, 57, 73, 97. (Received August 10, 1948.)

49*t*. V. E. Dietrich and Arthur Rosenthal: *Transcendence of factorial series with periodic coefficients.*

It is well known that every real number α can be represented in the form $\alpha=\sum_{n=1}^{\infty} a_n/n!$ where the a_n are integers and $0\leqq a_n<n$ (for $n=2, 3, \cdots$). In this note it is proved that α is a transcendental number if the a_n are periodic (except for the trivial case where almost all a_n are zero). More generally it is proved: If the power series $\phi(z)=\sum_{n=1}^{\infty}(a_n/n!)z^n$ has algebraic coefficients a_n (not almost all of them being zero) which form a periodic sequence, then $\phi(z)$ is a transcendental number for every algebraic z $(\neq 0)$. (Received October 12, 1948.)

50. Frank Harary: *The structure of finite Boolean-like rings.*

Boolean-like rings (BLR's) were defined by A. L. Foster as the class of rings in which the ring and logical operations are interdefinable through the same equations as for Boolean rings. In this paper, all finite BLR's are classified into isomorphism types by means of a canonical form for their multiplication tables. This canonical form is the basis for all further results on finite BLR's. Indecomposable BLR's are characterized as BLR's whose Boolean subring contains exactly two elements. It is verified that each finite BLR is isomorphic to the direct product of indecomposable BLR's. Not all BLR's have unique prime factorization. A necessary and sufficient condition for a finite BLR to have unique prime factorization is obtained. (Received October 15, 1948.)

ANALYSIS

51*t*. R. P. Boas: *The Charlier B-series.*

The B-series has the form $f(x) \sim \sum_{n=0}^{\infty} a_n \Delta^n \theta(x)$, where Δ denotes a receding difference; when $f(x)$ is defined only for integral values of x, $\theta(x) = e^{-\lambda}\lambda^x/x!$ for $x=0, 1, 2, \cdots$; $\theta(x)=0$ for $x=-1, -2, \cdots$; when $f(x)$ is defined for all real x, $\theta(x) = (2\pi)^{-1}e^{-\lambda}\int_{-\pi}^{\pi} e^{-ixu} \exp(\lambda e^{iu})du$. This paper develops an L^2 theory of the series, considering the meansquare approximation to a given $f(x)$ by sequences $\sum_{n=0}^{N} a_n^{(N)}\Delta^n\theta(x)$, where the $a_n^{(N)}$ are not necessarily determined by the usual formulas for the coefficients in the B-series. The resulting expansion theory applies to more general functions $f(x)$ than can be represented by the B-series itself. More general series of similar character are also considered. (Received October 11, 1948.)

52t. L. W. Cohen and Casper Goffman: *The completion of ordered fields.*

The problem dealt with is that of defining complete extensions of an ordered field by the methods of Dedekind and Cantor. The equivalence of these methods in the classical completion of the rational field suggests the possibility of their equivalence in general even if the field does not enjoy the property of Archimedes. Let $\mathcal{F}$ be an ordered field. By the method of Dedekind is meant the definition of the field $\mathcal{F}_D$ whose elements are those lower segments L with the property that if $g>\theta$, θ the zero in $\mathcal{F}$, there is an $f \in L$ such that $f+g \notin L$. By the method of Cantor is meant the definition of the field F_C whose elements S are the classes of mutually equivalent fundamental sequences in the topology of $\mathcal{F}$. The principal result obtained is that $\mathcal{F}_D$ and $\mathcal{F}_C$ are isomorphic complete extension of $\mathcal{F}$. Completeness in the sense of Archimedes is considered and the relation between the two concepts of completeness is indicated. The Hahn fields are examples. (Received October 10, 1948.)

53t. H. V. Craig and W. T. Guy: *On Jacobian extensors.*

The purpose of this paper is to introduce the concept Jacobian extensor together with certain features of the accompanying theory. The transformation equation involved differs from that of ordinary extensors by the presence of the coefficients X sup α inf ρ, which are equal to the binomial coefficient ${}_\alpha C_\beta$ times the derivative of order $\alpha-\rho$ of the weighted Jacobian, $(x/\bar{x})^w$. Rules are given for constructing Jacobian extensors by differentiation of weighted scalars and tensors, and it is found that the process extensive differentiation carries over to the new theory. A Jacobian connection is developed from the ordinary connection and by extensive differentiation extended to higher orders of M. By means of the extended Jacobian connection, the higher order intrinsic derivatives of weighted scalars and tensors are expressed as contractions. (Received October 15, 1948.)

54. O. W. Rechard: *A note on the representation of real numbers.*

Let p be any integer $\geqq 2$. For every continuous strictly increasing function $f(x)$ on $[0, p]$ to $[0, 1]$ Everett (*Representations for real numbers*, Bull. Amer. Math. Soc., vol. 52 (1946)) has defined an algorithm which associates with every real number between 0 and 1 a sequence of integers mod p. For the case $f(x)=x/p$ this algorithm yields the decimal representation of a number to the base p. This note contains, among other things, a characterization of the class of one-one correspondences between real numbers and sequences of integers mod p which can be obtained by applying such an algorithm. It is simply the class of those correspondences which can be obtained from the class of all continuous strictly increasing functions $F(x)$ on $[0, 1]$ to $[0, 1]$ by defining the sequence corresponding to the number x to be the sequence of integers in the non-$(p-1)$-terminating decimal expansion of $F(x)$ to the base p. As a result of this characterization, for example, there exists a continuous strictly increasing function $f(x)$ on $[0, 2]$ to $[0, 1]$ which distinguishes between algebraic and transcendental numbers in the sense that a number α between 0 and 1 is algebraic if and only if $\alpha = f(a_1 + f(a_2 + \cdots + f(a_n)) \cdots)$ for some finite sequence of integers $a_1, a_2, \cdots, a_n$, with $a_j = 0$ or 1. (Received October 15, 1948.)

55. W. T. Reid: *Symmetrizable completely continuous linear transformations in Hilbert space.*

This paper treats the spectral theory of a completely continuous linear transformation K in a complete (not necessarily separable) abstract Hilbert space $\mathfrak{H}$, where K is (left) symmetrizable in the sense that there exists a non-negative symmetric bounded transformation S on $\mathfrak{H}$ such that SK is symmetric. It is proved that K has a real nonzero proper value in case $SK \neq 0$; certain expansion theorems are established for transformations K such that $(Su, u) \neq 0$ for each proper element u of K corresponding to a nonzero proper value. Special attention is given to symmetrizable transformations corresponding to the vector integral equations which are equivalent to self-adjoint boundary problems satisfying a condition of definiteness as formulated by Bliss (Trans. Amer. Math. Soc. vol. 44 (1938) pp. 413–428) or by the author (Trans. Amer. Math. Soc. vol. 52 (1942) pp. 381–425). The method of proof is based on extremizing processes. In particular, the paper extends the results of Zaanen (Nieuw Archief voor Wiskunde (2) vol. 22 (1943) pp. 57–80; Nederl. Akademie van Wetenschappen, Proceedings vol. 49 (1946) pp. 194–204; Indagationes Math. vol. 8 (1946) pp. 91-101—see also his further papers in vols. 49, 50 of the same journal on the theory of linear integral equations). (Received October 15, 1948.)

56t. I. E. Segal: *Invariant measures on locally compact spaces.*

The existence and uniqueness of Haar measure are generalized by showing that an equicontinuous group G of unimorphisms of a uniformly locally compact space M leaves invariant a nonzero regular measure on M, which is unique (within proportionality) if and only if there is a point in M whose orbit is dense. If G is locally compact and acts continuously on M (that is, if the function on $G \times M$ to M describing the action of G is continuous) and if $f \in L_1(G)$ and $g \in L_\alpha(M)$, then $\int_G f(a) g(a^{-1}p) da$ exists for almost all p, da being the element of Haar measure, and defines a function in $L_\alpha(M)$ $(1 \leqq \alpha \leqq \infty)$. (Received October 1, 1948.)

Applied Mathematics

57. M. Z. Krzywoblocki: *On the theory of the limiting lines in compressible, rotational, inviscid, isentropic flow.*

Tollmien and Ringleb created the theory of the so-called "limiting lines" in compressible, inviscid, isentropic irrotational flow by a transformation of the physical plane onto the hodograph plane. In the present paper a similar procedure is applied to a compressible, inviscid, isentropic, rotational flow. By a suitable selection of the "generalized" velocity potential function a simple hodograph transformation is obtained. The condition of the vanishing functional determinant gives the equation of the "limiting lines". (Received September 23, 1948.)

58t. H. E. Salzer: *Formulas for numerical integration of first and second order differential equations in the complex plane.*

For numerical integration of a first or second order differential equation in the complex plane, wherever the solution is analytic, it is more natural and advantageous to choose $z=x+iy$ as the variable, rather than to express the equation as a simultaneous real system in x and y. Also, using z as the variable permits a closer choice of the fixed points upon which to base the approximating polynomials for the integrand. Integration formulas are given for a Cartesian grid. They were found by integrating the Lagrangian interpolation polynomial for three to nine points $z_j \equiv z_0+jh$, where h is the length of a square in the grid, and j is a complex integer j_1+ij_2. The configurations of z_j were chosen from considerations of convenience and closeness. A number of formulas for complex extrapolation are given, because they have an essential role in the integration process. All formulas are given with exact coefficients. A simple procedure for systematic integration, with suggested variations, is outlined for general use, suited to large-scale automatic computation devices. For configurations of z_j that are symmetric about the 45° ray, there are proved several useful relations among the coefficients occurring in the integration formulas. (Received October 8, 1948.)

Geometry

59t. V. G. Grove: *On the R_λ-associate of a line.*

In this paper the author extends a notion of Popa applied to a plane net to the asymptotic net on a surface. Thereby new geometric characterizations of Bell's R_λ-associate of a line and of the R_λ-derived curves are found. Applications give characterizations of the Darboux and Segre tangents and of the tangents of the Darboux-Segre pencil of conjugate nets. (Received October 15, 1948.)

Topology

60t. E. G. Begle: *The Vietoris mapping theorem for bicompact spaces.*

The Vietoris mapping theorem is proved for bicompact spaces. For any covering M of a space X, denote by $X(M)$ the complex consisting of all simplexes of X for each of which there is a set of M containing all its vertices. Then the statement of the theorem is: Let f be a mapping of X onto Y such that for each covering M of X, each point y of Y, and each integer k, $0 \leqq k \leqq n$, there is a refinement N of M such that any finite k-cycle on $X(N) \cap f^{-1}(y)$ bounds on $X(M) \cap f^{-1}(y)$. Then the homomorphism of the n-dimensional homology group of X into that of Y which is induced by f is an isomorphism and is onto. The coefficient group is arbitrary. If the coefficient group is compact or a field, then the hypothesis may be replaced by the weaker condition that the k-dimensional homology groups of $f^{-1}(y)$, $0 \leqq k \leqq n$, vanish for every y. For integer coefficients the theorem is not true with this weaker hypothesis, even in the compact metric case. (Received October 6, 1948.)

61. R. H. Bing: *Complements of continuous curves.*

Suppose space is compact, connected, locally connected, and metric. It is shown that each pair of points lies on an arc each of whose complementary domains has property S. If no pair of points separates space, the arc may be so chosen that its complement is also connected. If W is a closed totally disconnected point set, there is a dendron containing W such that each complementary domain of W has property S. (Received October 14, 1948.)

62t. Ky Fan: *Partially ordered additive groups of continuous functions*. I.

Let $C(\Omega)$ be the partially ordered additive group (p.o.a.g.) of all real continuous functions on a compact Hausdorff space Ω. A subgroup G of $C(\Omega)$ is called a characterizing subgroup, if: (1) G contains all constant functions; (2) for $x_0 \in \Omega$ and f, $g \in G$ such that $f(x_0) = g(x_0) = 0$, there is an $h \in G$ with $h \geqq f$, $h \geqq g$, $h(x_0) = 0$; (3) for any two points $x_1 \neq x_2$ of Ω, there is an $f \in G$ with $f(x_1) \neq f(x_2)$. A subgroup G of $C(\Omega)$ verifying (1), (2) is called a weakly characterizing subgroup. Let Ω, Ω^* be two compact Hausdorff spaces and G^* be a characterizing subgroup of $C(\Omega^*)$. Then Ω and Ω^* are homeomorphic, if and only if G^* is isomorphic (as p.o.a.g.) to a characterizing subgroup of $C(\Omega)$. More generally, Ω^* is a continuous image of Ω, if and only if G^* is isomorphic to a weakly characterizing subgroup of $C(\Omega)$. It is to be noticed that, when $C(\Omega)$ is regarded as a lattice, a characterizing subgroup of $C(\Omega)$ is not necessarily a sublattice of $C(\Omega)$. On the other hand, when $C(\Omega)$ is considered as a ring, a subring G of $C(\Omega)$ verifying (1), (3) fails to determine the topology of Ω. (Received October 14, 1948.)

63t. Ky Fan: *Partially ordered additive groups of continuous functions*. II.

Let G be a partially ordered additive group (p.o.a.g.). An element $f \in G$ is an Archimedean element of G if, for every $g \in G$, there is a natural number n with $nf \geqq g$. An element $f \geqq 0$ without this property is a non-Archimedean element of G. A subgroup H of G is singular in G, if every $f \in H$ is of the form $f = g - h$, where $g \in H$, $h \in H$ are non-Archimedean elements of G. A subgroup H of G is convex in G, if $H \neq G$ and if $h_1 \in H$, $h_2 \in H$, $h_1 \leqq f \leqq h_2$, imply $f \in H$. The following theorem is proved: A p.o.a.g. G is isomorphic to a characterizing subgroup of $C(\Omega)$ (cf. the preceding abstract) for a topologically unique compact Hausdorff space Ω, provided G satisfies these four conditions: (i) G has a subgroup R which is isomorphic to the simply ordered additive group of all real numbers; (ii) at least one element of R is an Archimedean element of G; (iii) if $nf + g \geqq 0$ for all natural numbers n, then $f \geqq 0$; (iv) if a maximal convex subgroup M of G is not maximal singular in G, then there are a finite number of elements $f_1, \cdots, f_n$ such that $f_i \notin M$ $(1 \leqq i \leqq n)$ and that each maximal singular subgroup of G contains at least one of them. By adding a fifth condition, a characterization of the entire group $C(\Omega)$ is also obtained. (Received October 14, 1948.)

64t. M. K. Fort: *A note on equicontinuity*.

It is proved in this note that the center of every algebraically transitive group of homeomorphisms acting on a compact metric space is equicontinuous. (Received August 12, 1948.)

65t. M. K. Fort: *A proof that the group of all homeomorphisms of the plane onto itself is locally arcwise connected*.

Let P be a plane and let $H(P)$ be the group of all homeomorphisms of P onto itself, topologized by defining convergence to mean uniform convergence on each compact subset of P. It is shown that corresponding to each positive number ϵ there is a positive number δ such that for each element f of $H(P)$ which is in the δ-neighborhood of the identity element I there is an isotopy F such that $F_0 = f$, $F_1 = I$ and F_t is in the ϵ-neighborhood of I for $0 \leqq t \leqq 1$. It follows that f and I can be joined by an arc inside the ϵ-neighborhood of I. Since $H(P)$ is a topological group, this proves that $H(P)$ is locally arcwise connected. (Received September 16, 1948.)

66*t*. Deane Montgomery: *Theorems on the topological structure of locally compact groups.*

Let G be a locally compact n dimensional group. If U is a neighborhood of the identity there exists a neighborhood V of the identity such that any n or $n-1$ cycle (reals mod one) in $\overline{V}$ bounds in $\overline{U}$. An analogous property is true for homogeneous spaces. If H is a closed subgroup of dimension $n-1$ and if dim G/H is finite, then dim $G/H=1$. If G is locally connected and A is a closed n-dimensional subset then A must contain an inner point; this also holds for homogeneous spaces. (Received October 4, 1948.)

67*t*. G. T. Whyburn: *Equicontinuous collections of mappings.*

If A and B are metric spaces, a collection G of mappings of A into B is called uniformly interior on $A_0 \subset A$ provided that for $e>0$ a $d>0$ exists such that for any $x \in A_0$ and $f \in G$, $f[V_e(x)] \supset V_d[f(x)]$. Similarly, G is uniformly light on A_0 provided that for any $e>0$ a $d>0$ exists such that if V is any connected set in A intersecting A_0 and of diameter $\geqq e$ and $f \in G$, then $\delta[f(V)] \geqq d$. If G is equicontinuous, A is locally compact and B is compact, then G is uniformly light on the compact subsets of A if and only if the limit mapping of every sequence uniformly convergent on a compact subset of A is light. If G is an equicontinuous collection of strongly interior mappings of a locally connected generalized continuum A into a locally connected continuum B and if the limit mapping of every sequence in G uniformly convergent on a compact subset A_0 of A is light, then G is uniformly interior on A_0. A uniformly bounded collection G of analytic functions on a region A is uniformly interior or light on a compact region $\overline{R}$ in A if and only if for some $d>0$, $\max_{z \in R} |f'(z)| \geqq d$ for all $f \in G$. (Received October 2, 1948.)

68*t*. G. T. Whyburn: *Normal regions and developability.*

Given locally connected generalized continua A and B and a light interior mapping $f(A)=B$, a region R in A is normal provided $f|\overline{R}$ is interior and f permutes with the boundary of R, that is, $f[F(R)]=F[f(R)]$. (This is similar to but not identical with Stoilow's notion of normal region.) If K is a continuum in B, H is a compact component of $f^{-1}(K)$ and $\epsilon>0$, there exists a normal region R satisfying $H \subset R \subset V_\epsilon(H)$. If A is the union of a strictly monotone increasing sequence of conditionally compact normal regions, f is said to be developable. This property holds if and only if each continuum in A lies in a conditionally compact normal region. Also, f is developable if and only if it is expansive in the sense that A is the union of a strictly monotone increasing sequence of conditionally compact regions $[R_n]$ such that no given compact set in B intersects the images of the boundaries of infinitely many of the R_n. (Received October 2, 1948.)

J. W. T. Youngs,
Associate Secretary