Logic and Foundations

339. David Nelson: *Constructible falsity.*

A definition for constructible truth of number theoretic statements is presented which entails the truth of a statement of the form "Not for all x, $A(x)$" only in case there is an effective method of constructing a natural number n such that "$A(n)$" is false. The definition is similar to that of Kleene (Journal of Symbolic Logic vol. 10 (1945) pp. 109–124) for realizability. Using results of this paper and Nelson (Trans. Amer. Math. Soc. vol. 61 (1947) pp. 307–368) a simply consistent system of number theory satisfying the truth definition is constructed. While some classically acceptable principles which are intuitionistically invalid are reinstated, the system becomes inconsistent upon the adjunction of an axiom schema representing the principle of contradiction. The system contains the systems of classical and intuitionistic number theory on suitable reinterpretation of the logical symbols of those systems. (Received July 28, 1947.)

340. Ira Rosenbaum: *A simple process for obtaining a formula satisfying the nth q-ary truth table of m-valued logic.*

The number, n, of any q-ary truth table of m-valued logic is determinable—by the methods of Bull. Amer. Math. Soc. Abstract 53-5-265—from the relation $n-1=\sum_{j=0}^{c-1}W_{c-j}\cdot m^{c-1-j}$ where $c=m^q$ and $W_i=$ the ith truth-value of the given table minus 1. Divide the summands above into m groups of m^{q-1} summands, and reduce all exponents to the remainders obtained on division of these exponents by m, adding 1 to each subsum obtained. Let the resulting sums be denoted by n_i, $i=1, 2, \cdots, m$. Then the following relation is provable: $F_n^{q(m)}(p_1, \cdots, p_q)\equiv\prod_{i=1}^{m}\{i(p_q)\supset F_{n_{m-i+1}}^{q-1(m)}(p_1, \cdots, p_{q-1})\}$ where the left member of the equivalence denotes the nth q-ary truth-function of m-valued logic, $i(p_q)$ denotes a singular function of p_q which has the value 1 when p_q has the value i and the value m otherwise. This relation yields a formula—defined in terms of &, $\supset$, singular functions and $(q-1)$-ary functions—which satisfies the nth q-ary truth table. Recursion on q finally yields a formula with the same property—built only from &, $\supset$, and singular functions—which is, for q or $m\geqq 3$, more easily obtained and comprehensible than conjunctive or disjunctive normal forms. (Received July 22, 1947.)

Statistics and Probability

341. H. D. Brunk: *The strong law of large numbers.*

Sufficient conditions for the strong law of large numbers for sequences of independent random variables involving moments of arbitrary even order are obtained using a method based on Kolmogoroff's. These are given for the strong law in a generalized form analogous to Feller's generalized form of the weak law (see Bull. Amer. Math. Soc. vol. 51 (1945) p. 827). Similar sufficient conditions are also given for sequences of independent random variables for which the moments do not exist. (Received May 12, 1947.)

342. J. L. Doob: *Asymptotic properties of Markoff transition probabilities.*

Let $P^{(n)}(x, A)$ be the probability of a transition from a point x into a set A (Markoff process). It is supposed that there is a self-reproducing distribution, that is,